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TEAM ROUND / 1 HOUR / 210 POINTS
October 2, 2010

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 70 points, for a total of 210 points.

Problem 1 (Prüfer Airlines). In a certain small European country there are only 5 cities. An airline decides to connect them by 4 (two-way) connections, so that it would be possible to fly from any city to any other, possibly with stops.

- (a) (35 points) In how many ways is it possible to do this?
- (b) (35 points) Same question but for 6 cities and 5 connections.

Problem 2 (Let's be friends). One hundred (100) people go through the following procedure. One-by-one, they each randomly point at a person who is not yet pointed at. A person may point at himself, so for example, the first person points at himself with probability 1%.

What is the probability that after this procedure there exist 75 people P_1, P_2, \dots, P_{75} so that person P_1 points at P_2 , person P_2 points at P_3 , \dots , person P_{74} points at P_{75} , and finally person P_{75} points at P_1 ?

Problem 3 (A very fair division). It is possible to divide the integers $1, 2, \dots, 8$ into two sets A and B in a unique manner so that

- 1 is in A ,

- A and B contain the same number of elements, and
- the sum of the elements in A equals the sum of the elements in B , and
- the sum of the squares of the elements in A equals the sum of the squares of the elements in B .

It is also possible to divide the integers $1, 2, \dots, 16$ into two sets A and B in a unique manner so that all of the above hold, *as well as*

- the sum of the *cubes* of the elements in A equals the sum of the *cubes* of the elements in B .

This is a two-part problem:

- (35 pts) What is the set A in the case of 8 numbers?
- (35 pts) What is the set A in the case of 16 numbers?

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Team ID:

Team name:

Answer 1:

- (a)
- (b)

Answer 2:

Answer 3:

- (a)
- (b)