1. Show that the mean $\mu$ of a random variable $X$ has the property

$$\min_c E(X - c)^2 = E(X - \mu)^2 = V(X).$$

2. (a) Prove that for any r.v. $X$

$$E|X| = \int_0^\infty P(|X| \geq t)dt.$$

(b) Given a square integrable r.v. $X$, show that for $\lambda \geq 0$,

$$P(X - E X \geq \lambda) \leq \frac{\sigma^2(X)}{\sigma^2(X) + \lambda^2}.$$

3. Show that random variables $X_n$, $n \geq 1$, and $X$ satisfy $X_n \rightarrow X$ in distribution iff

$$E[F(X_n)] \rightarrow E[F(X)]$$

for every continuous distribution function $F$.

4. If $\{X_n\}$ are iid r.v.s, then $E|X_1| < \infty$ if and only if $\sum_{n=1}^\infty X_n \frac{\sin n t}{n}$ converges a.s. for every $t \in (-\infty, \infty)$.

5. Let $\{X_n\}$ be a sequence of independent random variables.
   (a) If $EX_n = 0$ for $n = 1, 2, \ldots$, and $\sum_{n=1}^\infty \text{var}(X_n) < \infty$, show that $\sum_{n=1}^\infty X_n$ converges a.s.
   (b) State (without proof) Levy's inequality and use it to prove that $S_n = \sum_{k=1}^n X_k$ converges a.s. if and only if it converges in probability.