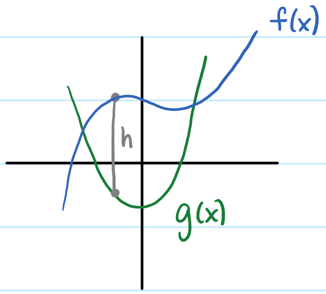
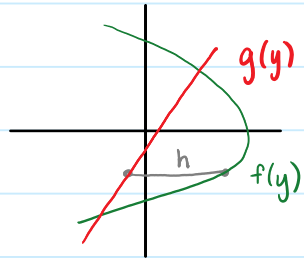
Chapter 6 Study Guide

Finding the distance between two curves





$h(x) = f(x) - g(x)$

- Read the graph from top to bottom

$h(y) = f(y) - g(y)$

- Read the graph from right to left

Finding volume using perpendicular cross sections

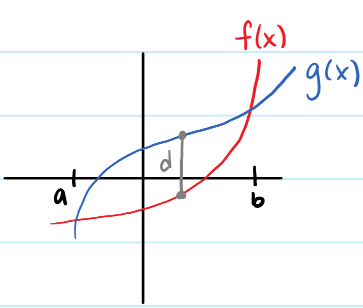
- Cross sections are made up of common geometric shapes with a main attribute corresponding the distance across the defined region.

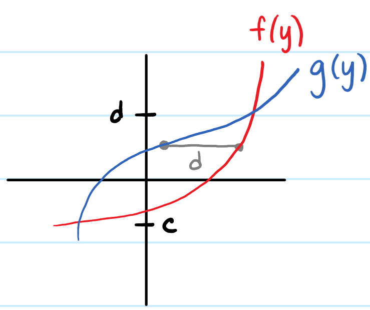
  Ex: Squares (side or diagonal)
  Circles / semi-circles (diameter)
  triangles (base or height)

  These are just a few possibilities. Read the problem thoroughly.

- Nothing is being revolved or rotated
- Volume is calculated:

$$V = \int_a^b A(x)\,dx \quad \text{or} \quad V = \int_c^d A(y)\,dy$$





$d = g(x) - f(x)$

$d = f(y) - g(y)$

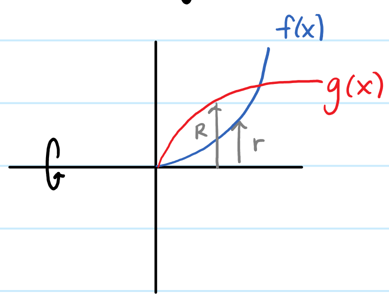
Volumes of revolution

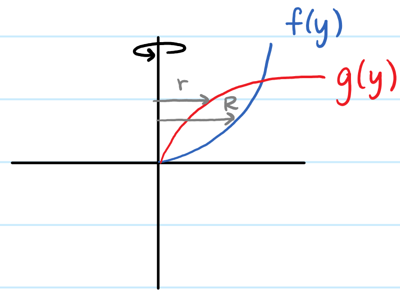
Washer/Disk Method - A Game of Radii

- Solids are created by revolving a region around an axis.
- Perpendicular cross sections are solid disks (no gap) or washers (mind the gap)
- For disks, $r(x) = 0$
  - about x-axis
  - about y-axis

$$V = \pi \int_a^b \left[R^2(x) - r^2(x)\right] dx \quad \text{or} \quad V = \pi \int_c^d \left[R^2(y) - r^2(y)\right] dy$$

- Radii are <u>always</u> perpendicular to the axis of rotation



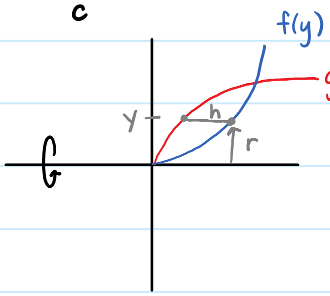


$R(x) = g(x)$, $r(x) = f(x)$ $\qquad$ $R(y) = f(y)$, $r(y) = g(y)$

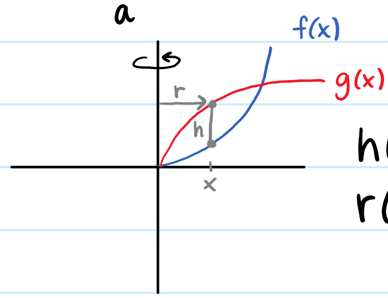
Shell Method - Adding up Rectangles

- Solids are created by revolving a region about an axis.
- Integrate with respect to the opposite variable as Washer/Disk
  - about x-axis
  - about y-axis

$$V = \int_c^d 2\pi R(y) h(y)\, dy \qquad V = \int_a^b 2\pi R(x) h(x)\, dx$$



$h(y) = f(y) - g(y)$
$r(y) = y$

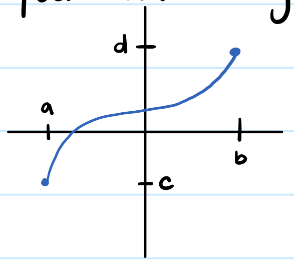


$h(x) = g(x) - f(x)$
$r(x) = x$

- heights are parallel to the axis of rotation, radii are still perpendicular

Arclength - How long is that line?

- Nothing is being revolved or rotated
- You can integrate with respect to x or y.



$$L = \int_a^b \sqrt{1+f'(x)^2}\,dx = \int_c^d \sqrt{1+g'(y)^2}\,dy$$

Surfaces of Revolution

- Surfaces are formed by revolving a line around an axis.
- You can integrate with respect to x or y.
  - about x-axis
  - about y-axis

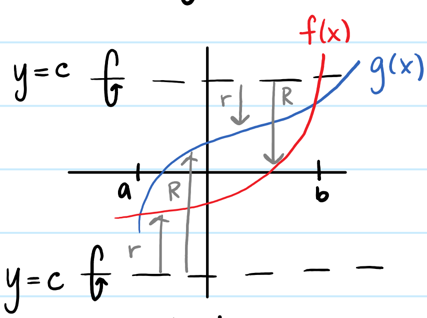
$$S = \int_a^b 2\pi f(x)\sqrt{1+f'(x)^2}\,dx \qquad S = \int_c^d 2\pi g(y)\sqrt{1+g'(y)^2}\,dy$$

$$= \int_c^d 2\pi y\sqrt{1+g'(y)^2}\,dy \qquad = \int_a^b 2\pi x\sqrt{1+f'(x)^2}\,dx$$

$$y = f(x), \quad \sqrt{1+f'(x)^2}\,dx = \sqrt{1+g'(y)^2}\,dy, \quad x = g(y)$$

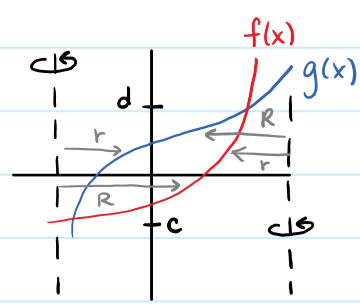
Using an axis of rotation other than the x or y-axis

- about $y=c$ (horizontal line)
- about $x=c$ (vertical line)



axis is above region:
$R = c - f(x)$
$r = c - g(x)$

axis is below region:
$R = g(x) - c$, $r = f(x) - c$



axis is to the right:
$R = c - g(x)$
$r = c - f(x)$

axis is left of region:
$R = f(x) - c$
$r = g(x) - c$

Work

The amount of energy required to move an object from point A to point B.

Constant Force

- If the weight (F) of an object is <u>unchanged</u> throughout the motion
  - Ex: lifting a bag of sand $W = mgh = (\text{weight})(\text{height})$
- If the force applied to an object is <u>unchanged</u> throughout the motion
  - Ex: Sliding a box across the floor $W = F \cdot d = (\text{force})(\text{distance})$

Variable Forces

- The amount of force is <u>changing</u> throughout the motion
- If the force is a well defined function, $W = \int_a^b F(x)\,dx$
  - Ex: Spring Force $F(x) = kx$
    - k = spring constant x = distance stretched or compressed

Gravitational Force

$$F = \frac{Gm_1m_2}{d^2} \rightarrow F(x) = \frac{C}{x^2}, \quad C = \text{constant}, \quad x = \text{distance between objects}$$

Electric Force

$$F = \frac{kq_1q_2}{d^2} \rightarrow F(x) = \frac{C}{x^2}, \quad C = \text{constant}, \quad x = \text{distance between objects}$$

- If the amount of weight or force varies as height varies, we have to define a function for F <u>and</u> d inside the integral.
  - Ex: Pumping water out of a tank $F = (\text{density})(\text{Area})\,dy$ $d = h - y$
  - Lifting a large cable or rope $F = \left(\frac{\text{weight}}{\text{length}}\right) dy$ $d = h - y$