Division of Academic Enhancement
UNIVERSITY OF GEORGIA

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MATH 1113 Exam 3 Review

Fall 2017

Topics Covered

Section 4.1: Angles and Their Measure

Section 4.2: Trigonometric Functions Defined on the Unit Circle

Section 4.3: Right Triangle Geometry

Section 4.4: Trigonometric Functions of Any Angle

Section 4.5: Graphs of Sine and Cosine Functions

Section 4.7: Inverse Trigonometric Functions

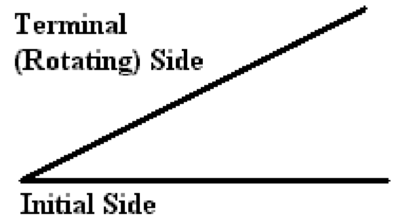
<u>Section 4.1: Angles and Their Measure</u>

<u>Trigonometric Functions of Acute Angles</u>

An <u>acute angle</u> is an angle with a measure satisfying:

$$0° \leq \theta < 90°$$

$$0 \leq \theta < \frac{\pi}{2}$$



An <u>obtuse angle</u> is an angle with a measure satisfying:

$$90° < \theta$$

$$\frac{\pi}{2} < \theta$$

A right triangle is a triangle with one 90° angle.

Angles are <u>complementary</u> when their sum adds up to 90°.
Angles are <u>supplementary</u> when their sum adds up to 180°.

<u>Radian Measure</u>

1 radian is the angle measure of the arc of a circle where $s = r$. $s = arclength, r = radius$

$$180° = \pi \ radians$$

To convert from radians to degrees:

$$\theta° \frac{\pi}{180°} = \theta \ (in \ radians)$$

To convert from degrees to radians:

$$\theta \frac{180°}{\pi} = \theta°$$

π ≈ 3.14 radians

<u>Radian Measure and Geometry</u>

$s = r\theta$ where $s = arclength, \theta = angle$ (in radians) and $r = radius$

Area of a sector of a circle: There are three possible ways to measure depending on which two variables you know. θ MUST BE IN RADIANS!!!

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} rs = \frac{s^2}{2\theta}$$

<u>Angular Speed</u>	<u>Linear Speed</u>
$\omega = \frac{\theta}{t}$	$v = \frac{d}{t}$

$$v = r\omega$$

Examples

1. Kim’s donut tasting party was so successful that it propelled her YouTube hits to such a level that she was invited to compete in a bake-off competition on the Food Network. Kim won first prize and was awarded The Golden Donut. She was so excited when she won that she dropped it and it rolled 100m before she could catch it. If the diameter of The Golden Donut trophy is 0.1m, answer the following:

   (a) Through what total angle did The Golden Donut rotate in radians?

   (b) Through what total angle did The Golden Donut rotate in degrees?

   (c) How many revolutions did The Golden Donut rotate? (Round your answer to 1 decimal place)

2. A circular sector with central angle 150° has an area of 20 square units. Determine the radius of the circle.

Section 4.2: Trigonometric Functions Defined on the Unit Circle

The Unit Circle

The unit circle consists of all points (x, y) that satisfy the equation $x^2 + y^2 = 1$ which is a circle of radius 1. See the back page for a blank unit circle.

Pythagorean identities

$\sin^2\theta + \cos^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$

Opposite Angle Identities

$\cos(-\theta) = \cos\theta$	$\sin(-\theta) = -\sin\theta$	$\tan(-\theta) = -\tan(\theta)$
Cosine is an even function	Sine is an odd function	Tangent is an odd function

Examples

3. In which quadrants to $\tan\theta$ and $\sin\theta$ have the same sign?

4. If $\cos x = \frac{\sqrt{2}}{2}$, find $\sin x$, $\cos(-x)$ and $\tan x$.

5. Assume $6\sin^2 x - 7\cos^2 x = 1$. Find the value of $\csc^2 x$.

6. Give the location(s) on the unit circle where the following quantities are undefined on the interval $[0,2\pi]$.

(a) $\sec\theta$

(b) $\csc\theta$

(c) $\tan\theta$

(d) $\cot\theta$

Section 4.3: Right Triangle Geometry

The following 6 Trig functions only apply to right triangles.

$$\cos\theta = \frac{adj}{hyp}$$

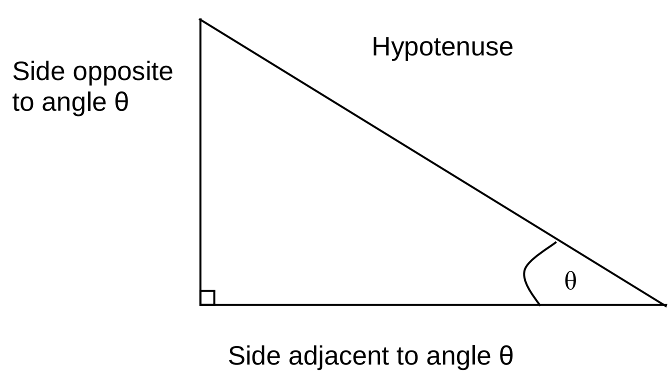
$$\sin\theta = \frac{opp}{hyp}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opp}{adj}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj}$$

$$\csc\theta = \frac{1}{\sec\theta} = \frac{hyp}{opp}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} = \frac{adj}{opp}$$



Examples

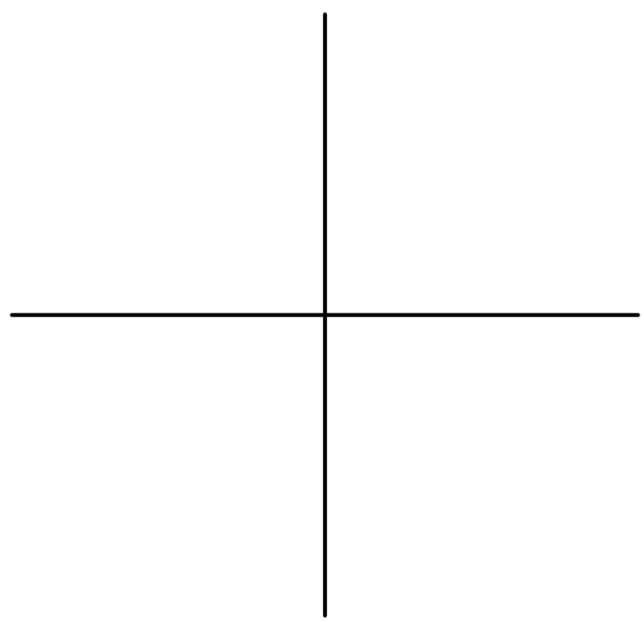
7. Assume θ lies in quadrant 3 and the terminal side of θ is perpendicular to the line $y = -x + 2$. Determine $\sin\theta$ and $\sec\theta$.

8. Let point $A(-3, -6)$ be the endpoint of the terminal side of an angle θ in standard position. Compute the following:

(a)

$\tan\theta =$

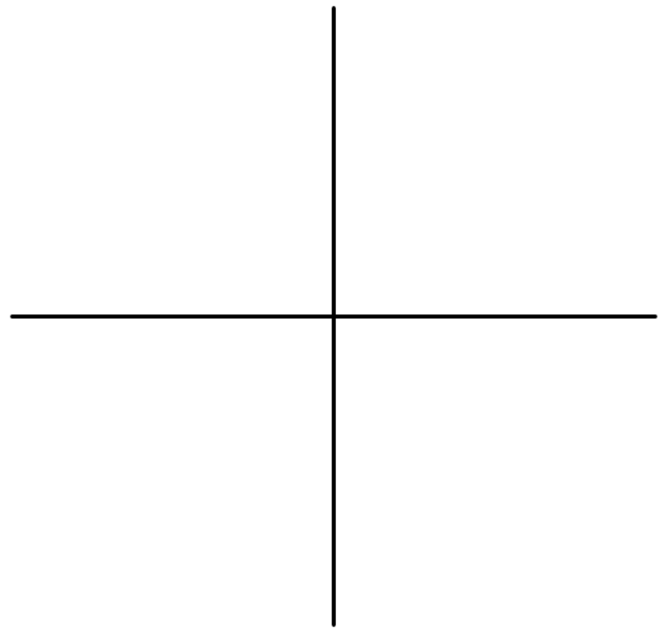
$\sec\theta =$



Let point $B(4, -7)$ be the endpoint of the terminal side of an angle α in standard position. Compute the following:

(b)

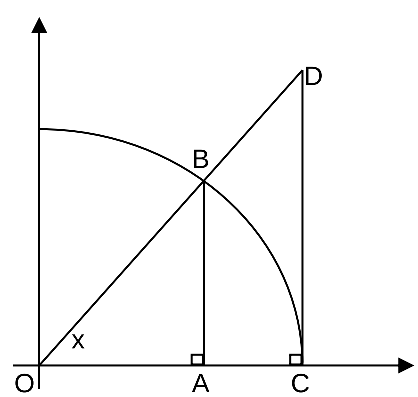
$\cot \alpha =$



$\sin \alpha =$

9. Find all possible values of $\sin x$ in the interval $[0, 2\pi]$ when $\cos x = 5/7$. There may be more than one right answer.

10. The arc in the graph is a section of the unit circle centered at $(0,0)$. Find the lengths of CD and OD in terms of x.



Section 4.4: Trigonometric Functions of Any Angle

Trigonometric Functions of Angles
Positive angles are measured from the positive x-axis rotating counterclockwise.
Negative angles are measured from the positive x-axis rotating clockwise.

Reference Angle is the acute angle ($\theta \leq \pi/2$) formed by the terminal side and the horizontal axis.

Coterminal angles are angles that share the same terminal side.

Examples

11. Find the reference angle for the following:

(a)

$$\theta = \frac{11\pi}{6}$$

(b)

$$\theta = 317°$$

12. Find all the exact values of x that satisfy $\tan x = 1$ on the interval $[-2\pi, 2\pi]$. Show your work or no credit will be awarded. (Hint: A diagram counts as work)

13. Devin was so upset about Florida's crushing loss to Georgia this season that he climbed a tree after the game and wouldn't come down. His friends Erik and Scotti need a ladder to retrieve him but don't know how tall the tree is. If Erik is standing to the left of the tree with an angle of elevation of 21° and his distance to the top of the tree is 42m. Scotti is standing to the right of the tree with an angle of elevation of 47°. Answer the following.

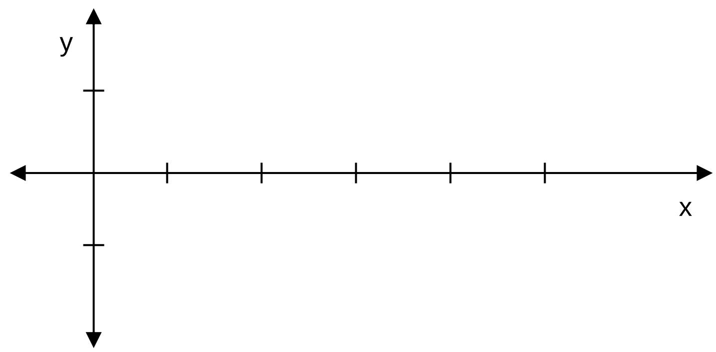
(a) How long does the ladder need to be to reach Devin if their plan is to climb straight up the tree?

(b) How far apart are Erik and Scotti from each other?

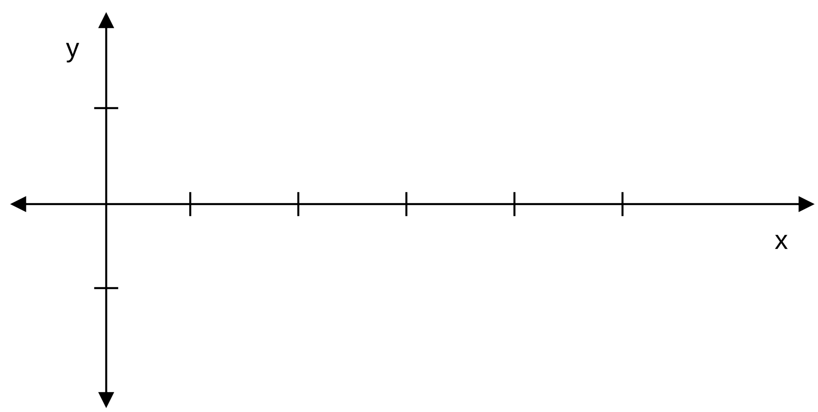
Section 4.5: Graphs of Sine and Cosine Functions

Graphs of Sine and Cosine Functions
Sketch the graphs of the 6 trigonometric functions in the space provided.

$\sin x$



$\cos x$



General Wave Properties
Waves oscillate, or bounce, between their maximum and minimum values.

$$y = A\sin[B(x - C)] + D \qquad y = A\cos[B(x - C)] + D$$

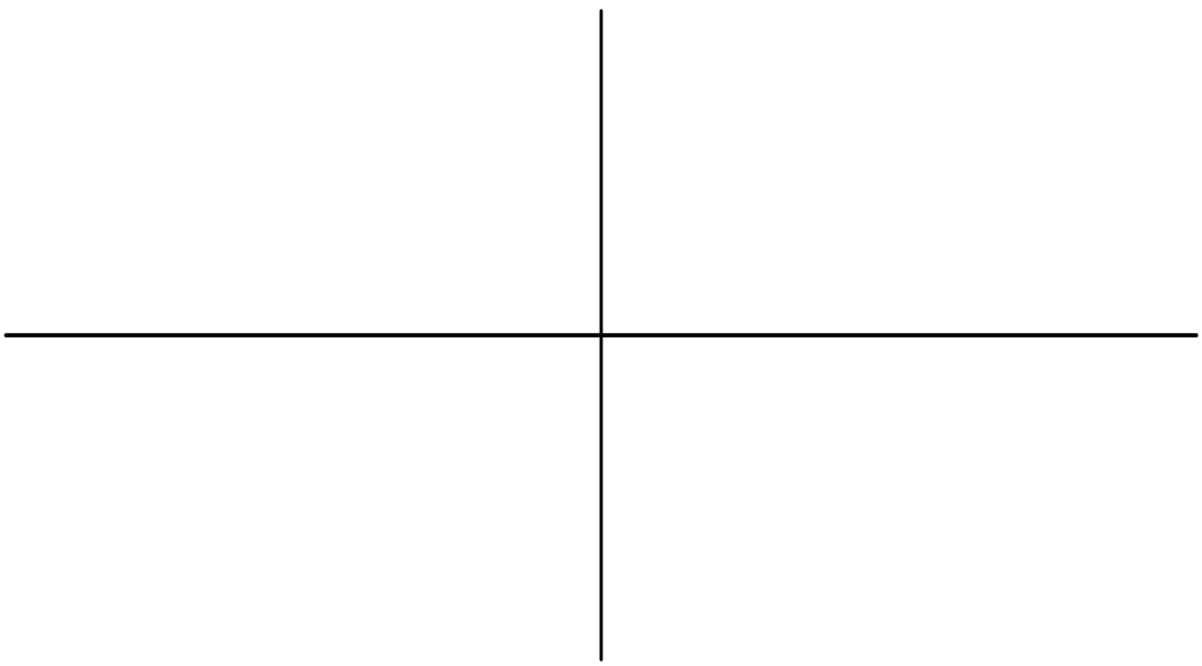
$$A = amplitude = \frac{max - min}{2} \qquad B = frequency = \frac{2\pi}{T}, T = time\ period$$

$$C = phase\ shift = horizontal\ shift \qquad D = vertical\ shift = \frac{max + min}{2}$$

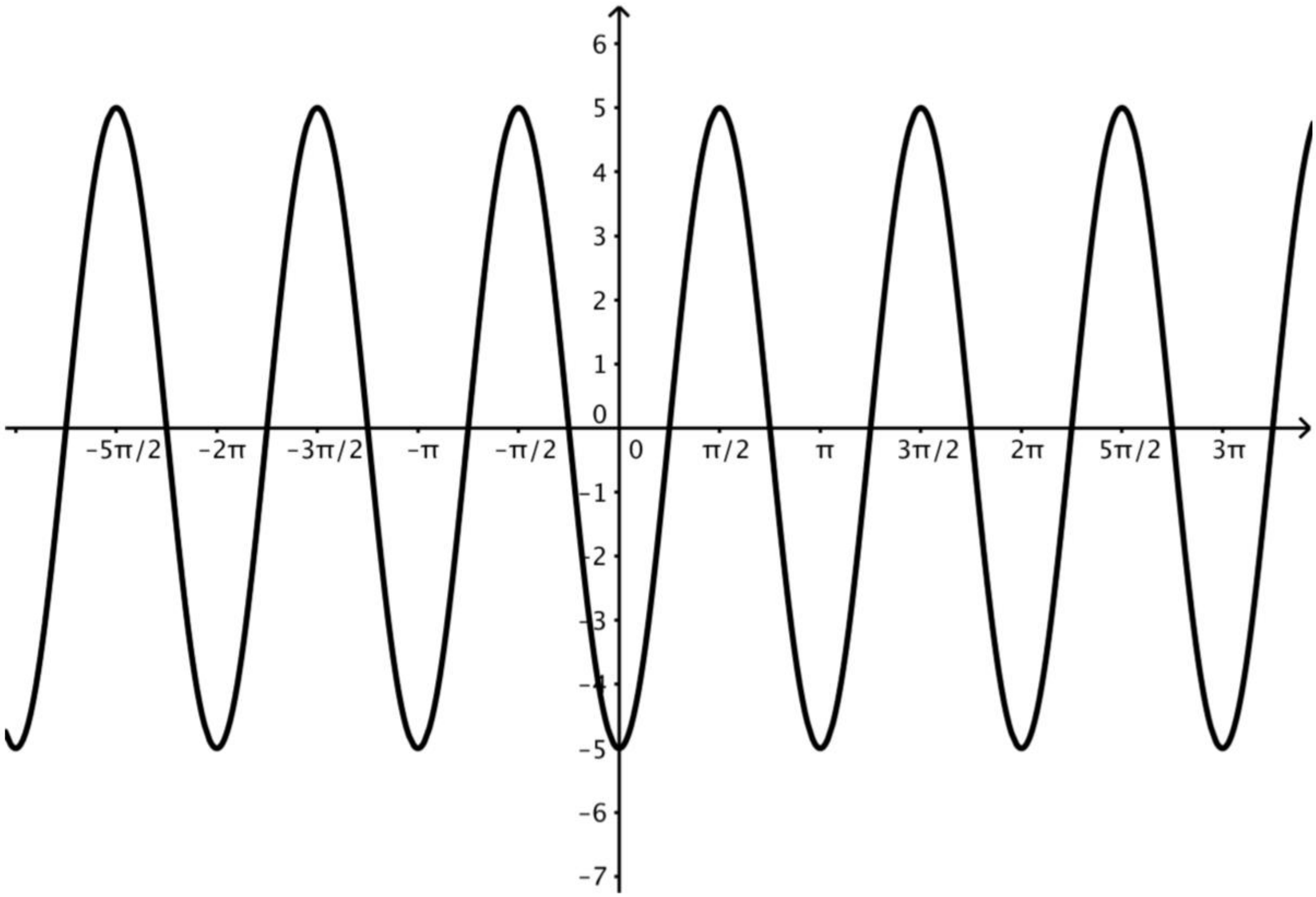
Examples

14. A sine wave oscillates between 6 and 10, has a period of 4π and is shifted left π.
    (a) Write the equation of the wave in the form $y = A\sin[B(x - C)] + D$.

    (b) Sketch the wave you found in part (a) on the interval $[-2\pi, 2\pi]$. Clearly label the amplitude, period, y-intercept and scale the x-axis by increments of $\frac{\pi}{2}$.



15. The graph below is a graph of a function $f(x) = a\cos(2x + c)$. Assuming that $a > 0$ and $c > 0$, determine the values of a and c that would produce the graph below.



Section 4.7: Inverse Trigonometric Functions

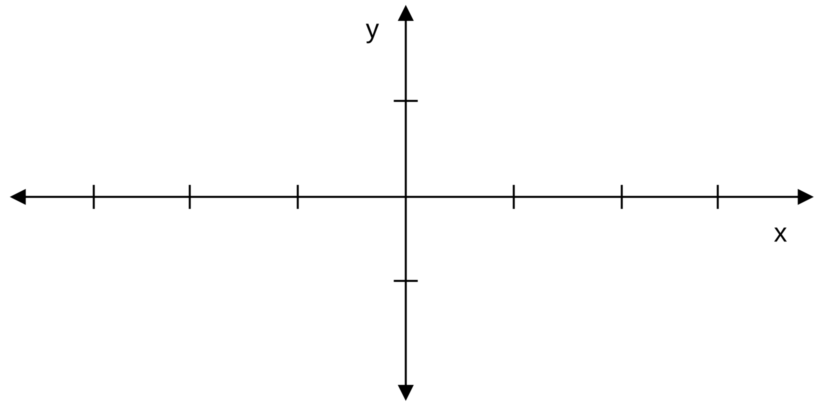
Inverse Trig Functions and their Graphs

Sketch the graphs of the inverse trig functions in the space provided. State their respective domain and range.

$\sin^{-1}(x)$

D:

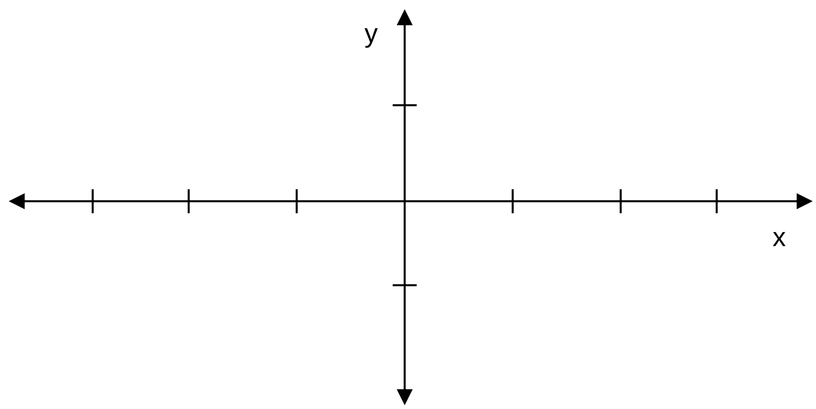
R:



$\cos^{-1}(x)$

D:

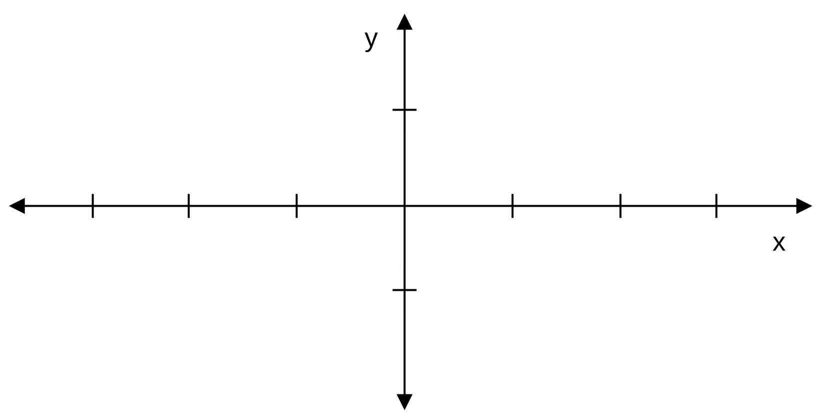
R:



$\tan^{-1}(x)$

D:

R:



Inverse cosine	arccos	the unique number in the interval $[0,\pi]$ whose cosine is x.
Inverse sine	arcsin	the unique number in the interval $[-\pi/2,\pi/2]$ whose sine is x.
Inverse tan	arctan	the unique number in the interval $[-\pi/2,\pi/2]$ whose tangent is x.

Properties of Inverse Trig Functions

$\sin(\sin^{-1} x) = x$	For every x in $[-1,1]$
$\sin^{-1}(\sin x) = x$	For every x in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos(\cos^{-1} x) = x$	For every x in $[-1,1]$
$\cos^{-1}(\cos x) = x$	For every x in $[0, \pi]$
$\tan(\tan^{-1} x) = x$	For all reals $(-\infty, \infty)$
$\tan^{-1}(\tan x) = x$	For every x in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Examples

16. Find the exact value of the following:

(a)

$$\sin\left(\arccos\left(-\frac{2}{3}\right)\right)$$

(b)

$$\arcsin\left(\sin\left(\frac{5\pi}{6}\right)\right)$$

17. Verify that the following expression is an identity.

$$\frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = 2\sec^2 x$$