1. Suppose \((X, d)\) is a metric space. State criteria for continuity of a function \(f : X \to X\) in terms of
   i) open sets,
   ii) \(\epsilon\)'s and \(\delta\)'s, and
   iii) convergent sequences.
Then prove that (iii) implies (i).

2. Prove that every compact, Hausdorff topological space is normal.

3. Write \(Y\) for the interval \([0, \infty)\), equipped with the usual topology. Find, with proof, all subspaces \(Z\) of \(Y\) which are retractions of \(Y\).

4. State, without proof, a criterion in terms of the fundamental group for a covering map \(p : \tilde{X} \to X\) to be regular. Give an example of a covering map that is not regular, and show explicitly how it fails to satisfy this criterion.

5. Start with the unit disk \(D^2\) and identify points on the boundary if their angles, thought of in polar coordinates, differ by a multiple of \(\frac{\pi}{2}\). Let \(X\) be the resulting space. Use van Kampen's theorem to compute \(\pi_1(X, *)\).

6. Let \(S^2 \to \mathbb{RP}^2\) be the universal covering map. Is this map null-homotopic? Give a proof of your answer.

7. Compute the homology of the space \(X\) obtained by attaching a Mobius band to \(\mathbb{RP}^2\) via a homeomorphism of its boundary circle to the standard \(\mathbb{RP}^1\) in \(\mathbb{RP}^2\).

8. Let \(X = S^2/\{p_1 = \cdots = p_k\}\) be the topological space obtained from the 2-sphere by identifying \(k\) distinct points on it \((k \geq 2)\).
   a) Find the fundamental group of \(X\).
   b) Find the Euler characteristic of \(X\).
   c) Find the homology groups of \(X\).

9. Let \(M\) and \(N\) be finite CW-complexes.
   a) Describe the cellular structure of \(M \times N\) in terms of the cellular structure of \(M\) and \(N\).
   b) Show that the Euler characteristic of \(M \times N\) is the product of the Euler characteristics of \(M\) and \(N\).