Division of Academic Enhancement
UNIVERSITY OF GEORGIA

Department of Mathematics
Franklin College of Arts and Sciences
UNIVERSITY OF GEORGIA

dae.uga.edu/tutoring/math-statistics

math.uga.edu/studyhall

MATH 1101 Exam 4 Review

Spring 2018

Topics Covered

Section 6.1 Introduction to Polynomial Functions
Section 6.2 The Behavior of Polynomial Functions
Section 6.3 Modeling with Polynomial Functions

What's in this review?

1. **Review Packet**
   The packet is filled in along with the instructor during the live review. If you missed the live review, see the video link at either web address at the top of the title page for the video link to watch the review.
2. **Useful Formulas**
   For your convenience, selected screen shots are available as a quick reference. The video shows the calculator screen for specific problems in the packet.
3. **Practice Problems**
   The problems are meant for you to try at home after you have watched the review. The answers are provided on the last page. There are tutors in Milledge Hall and Study Hall to help you if needed.

Section 6.1 Introduction to Polynomial Functions

General Form of a Polynomial Function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0$$

where a_n is the *leading coefficient* and n, the highest exponent, is the *degree* of the polynomial.

Examples of Polynomial Functions

A polynomial of degree $n = 1$ is a linear function: $f(x) = ax + b$

A polynomial of degree $n = 2$ is a quadratic function: $f(x) = ax^2 + bx + c$

A polynomial of degree $n = 3$ is a cubic function: $f(x) = ax^3 + bx^2 + cx + d$

A polynomial of degree $n = 4$ is a quartic function: $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

Properties of Quadratic Functions $f(x) = ax^2 + bx + c$

- The graph of a quadratic is a parabola
- The sign of the leading coefficient a indicates the direction in which the parabola opens
  - $a > 0$, the parabola opens up
  - $a < 0$, the parabola opens down
- The turning point of a parabola is called the vertex (h, k)
  - $h = -\frac{b}{2a}$
  - $k = f(h)$
- Domain is all real numbers $(-\infty, \infty)$
- The range of a parabola is
  - If $a > 0$, $R: [k, \infty)$
  - If $a < 0$, $R: (-\infty, k]$
- The zeros or solutions to a parabola are any values of x where $f(x) = 0$. There will be at max two values. These values are also x-intercepts.

Example 1

Answer the following about $f(x) = 2x^2 + 17x - 35$

(a) Find all x-intercepts. If there are two, write the factored form of the quadratic function.

(b) Find the vertex

(c) Find the domain and range of the function

(d) Find the intervals where the function is increasing or decreasing

(e) What is the concavity of the graph of f

Example 2

Answer the following about $f(x) = -2x^2 - 15x + 42$

(a) Find all x-intercepts. If there are two, write the factored form of the quadratic function.

(b) Find the vertex

(c) Find the domain and range of the function

(d) Find the intervals where the function is increasing or decreasing

(e) What is the concavity of the graph of f

Example 3

Find the maximum and minimum values of the function $f(x) = -0.15x^2 + 3.32x + 123.62$ on the following intervals

(a) $(-\infty, \infty)$

(b) $[5,15]$

(c) $[30,40]$

<u>Projectile motion</u>

A projectile can be modeled by a quadratic function using the following: $h(t) = -16t^2 + v_0 t + h_0$ where v_0 is the initial velocity and h_0 is the initial height.

Example 4

A rock is thrown straight up at a velocity of 50 ft/sec from the roof of a building that is 100 ft tall.

(a) When is the rock at a height of 130 ft?

(b) What is the maximum height the rock reaches?

(c) When does the rock reach max height?

(d) When does the rock hit the ground?

<u>Section 6.2 The Behavior of Polynomial Functions</u>

<u>Inflection Point/Concavity</u>
Inflection points occur where there is a change in concavity. We can identify inflection points by finding the location of the steepest slope (largest ARC magnitude) on the graph. There is always an inflection point somewhere between a maximum and minimum turning point of the graph.

On a <u>*cubic*</u> function, the inflection point occurs halfway between the two turning points (if they exist) and you can compute the location by averaging their respective x-values.

Once you identify the interval (a, b) in which the inflection point occurs, you can find it on the calculator by doing the following:

<u>Calculator</u>
1. Enter $f(x)$ in Y_1
2. Set $L_1 = seq(x, x, a, b, \Delta t)$
3. Set $L_2 = Y_1(L_1)$
4. Set $L_3 = \Delta List(L_2)/\Delta List(L_1)$
5. Find the value in L_3 with the largest magnitude
6. The interval on which they occur contains the x coordinate of the inflection point (L_1), and y-coordinate (L_2)

Example 5

$$g(x) = -12x^4 + 61x^3 + 525x^2 - 1001x - 4989$$

(a) Find all the x-intercepts

(b) Find all the local maximum and minimum points

(c) Find the domain and range

(d) Find the intervals where the function is increasing or decreasing

(e) Approximate the x-coordinate of all inflection points to one decimal place.

(f) Discuss the concavity of g

Example 6
Company A builds storage sheds of dimension 10 ft X 12 ft X 14 ft. Company B advertises that its sheds are better because they are x ft longer, wider and taller and can store 3 times the volume! What is the value of x? What are the dimensions of the shed that company B sells?

Section 6.3 Modeling with Polynomial Functions

Polynomial Regression
Based on multiple linear regression (Section 4.4) We use the following to model:

Calculator
1. Go to STAT→CALC→
2. LinReg($ax + b$) to build linear models (degree 1)
3. QuadReg to build quadratic models (degree 2)
4. CubicReg to build cubic models (degree 3)
5. QuartReg to build quartic models (degree 4)

Coefficient of Determination, R^2
To compare multiple models to find the best fit, we look at their respective R^2 values.
- R^2 measures the amount of explained error/variation in the model
- R^2 is a percentage in decimal for where $0 \leq R^2 \leq 1$
- $1 - R^2$ is the percentage of unexplained error or variation in the model
- The larger R^2 is, (closer to 1) the better fit it is to the data set

Example 16

Complete the table below for the following data set

x	1	2	3	4	5	6	7	8	9	10
y	32.2	201.4	489.1	1203.6	1904.5	2567.3	2999.1	2876.5	2057.2	152.9

Model Type	R^2	SSE
Linear		
Quadratic		
Cubic		
Quartic		

Which one is better? Why?

Useful Calculator Fucntions

<u>Finding the value of a function for a given x (input)</u>

<u>Option 1</u>

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Go to 2ND→CALC→VALUE. Enter the x value and hit enter.

<u>Option 2</u>

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Go to GRAPH→TRACE. You can move the cursor along the graph to see various points.

<u>Calculating a single ARC value from a given equation for an interval $[a, b]$</u>

<u>Option 1</u>

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Type $(Y_1(b) - Y_1(a))/(b - a)$. You can find Y_1 using VARS→Y-VARS→Function→ Y_1

For this method, pay close attention to your use of parenthesis!

<u>Option 2</u>

1. Go to the list editor STAT→Edit…
2. Set $L_1 = \{a, b\}$. The curly brackets { } can be found by 2ND→ ( )
3. Set $L_2 = Y_1(L_1)$. Use Y-VARS to grab Y_1 as described above
4. Set $L_3 = \Delta List(L_2)/\Delta List(L_1)$. You can find $\Delta List$ using 2ND→LIST→OPS→ $\Delta List$

<u>Finding the SSE in the Calculator</u>

1. Go to STAT→Edit…
2. Enter the x values in L_1
3. Enter the y values in L_2
4. Enter $Y_1(L_1)$ in L_3
5. Enter $L_2 - L_3$ in L_4
6. Enter L_4^2 in L_5
7. Go to 2nd →STAT→MATH→sum(L_5)

Additional Practice Problems

1. Find the exact solutions of $15x^3 - 49x^2 - 104x + 240 = 0$
2. The height in feet of a cannonball is given by the equation $h(t) = -16t^2 + 370t + 12$
   - What is the initial height and velocity of the cannonball?
   - What is the maximum height of the cannonball?
   - What time does the cannonball hit the ground?
3. Use the function $f(x) = 576x^4 + 120x^3 - 6650x^2 - 2375x + 6250$
   - What is the domain of f?
   - What is the range of f?
   - Find the local maximum.
   - Find the local minimums.
   - Where is this function increasing?
   - Where is this function decreasing?
   - Find the inflection points.
   - Describe the concavity.
   - Find the four zeros.
   - What is the factored form of this equation
4. A $2 \times 3 \times 7$ in. metal block is plated with zinc in such a way that each dimension is increased by the same amount, x. The new block has double the volume of the original. Find the value of x and the dimensions of the plated block.
5. Find the linear interpolating polynomial $f(x) = ax + b$ for the points (7,10) and (12,5)
6. Find the quadratic interpolating polynomial $f(x) = ax^2 + bx + c$ for the points (−2,−4), (0,6), and (3,−9)
7. Find the quartic interpolating polynomial $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ for the points (−2,−3), (−1,7), (0,5), (1,3), and (2,−11)
8. Use this data for the following questions.

x	1	2	3	4	5	6	7	8	9	10
y	39.9	196.5	567.1	1147.1	1856.7	2552.4	3027.1	2984.2	2075.9	148.5

   (a) Find the linear model that best fits this data, What is its R^2? What is its SSE?

   (b) Find the quadratic model that best fits this data, What is its R^2? What is its SSE?

   (c) Find the cubic model that best fits this data, What is its R^2? What is its SSE?

   (d) Find the quartic model that best fits this data, What is its R^2? What is its SSE?

   (e) Which model is the best fit?

Answers:

1. $x = -\frac{12}{5}, \frac{5}{3}, 4$
2. - $h_0 = 12$ ft., $v_0 = 370$ ft/s.
   - 2150.78 ft.
   - $t = 23.15$ s.
3. - $(-\infty,\infty)$
   - $(-16987.2,\infty)$
   - $(-0.18,6461.96)$
   - $(-2.39,-8903.67)$, $(2.41,-16987.2)$
   - $(-2.39,-0.18)$, $(2.41,\infty)$
   - $(-\infty,-2.39)$, $(-0.18,2.41)$
   - $x = -1.44$ and $x = 1.34$
   - Concave up, then concave down then concave up
   - $x = -\frac{25}{8}, -\frac{5}{4}, \frac{5}{6}, \frac{10}{3}$
   - $f(x) = 576(x+25/8)(x+5/4)(x-5/6)(x-10/3)$ or $f(x) = (8x+25)(4x+5)(6x-5)(3x-10)$
4. $x = .82 \rightarrow 2.82\text{in.} \times 3.82\text{in.} \times 7.82\text{in.}$
5. $f(x) = -1x + 17$
6. $f(x) = 2x^2 + 1x + 6$
7. $f(x) = -1x^4 + 0x^3 + 1x^2 - 2x + 5$
8. (a) $f(x) = 197.30x + 374.40$, $R^2 = .2521$, $SSE = 9525971$

   (b) $f(x) = -106.19x^2 + 1365.42x - 1961.87$, $R^2 = .7196$, $SSE = 3571576$

   (c) $f(x) = -33.60x^3 + 448.22x^2 - 1191.60x + 921.09$, $R^2 = .9933$, $SSE = 84263$

   (d) $f(x) = -2.20x^4 + 14.76x^3 + 94.34x^2 + -224.46 + x + 166.73$, $R^2 = .9996$, $SSE = 4674.1$

   (e) Quartic