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MATH 1101 Exam 3 Review - Gentry

Spring 2018

Topics Covered

Section 5.3 Fitting Exponential Functions to Data

Section 5.4 Logarithmic Functions

Section 5.5 Modeling with Logarithmic Functions

What's in this review?

1. **Review Packet**
   The packet is filled in along with the instructor during the live review. If you missed the live review, see the video link at either web address at the top of the title page for the video link to watch the review.
2. **Useful Formulas**
   For your convenience, selected screen shots are available as a quick reference. The video shows the calculator screen for specific problems in the packet.
3. **Practice Problems**
   The problems are meant for you to try at home after you have watched the review. The answers are provided on the last page. There are tutors in Milledge Hall and Study Hall to help you if needed.

<u>Section 5.3 Fitting Exponential Functions to Data</u>

Recall from Last Time…

<u>Exponential Functions</u>
A function is called exponential if the variable appears in the exponent with a constant in the base.

<u>Exponential Growth Function</u>
Typically take the form $f(x) = ab^x$ where a and b are constants.

- b is the base, a positive number > 1 and often called the *growth factor*
- a is the y intercept and often called the *initial value*
- x is the variable and is in the exponent of b

<u>Exponential Decay Function</u>
Typically take the form $f(x) = ab^x$ where a and b are constants.

- b is the base where $0 > b > 1$ and often called the *decay factor*
- a is the y intercept and often called the *initial value*
- x is the variable and is in the exponent of b

<u>How do we build models off data sets that have exponential growth or decay?</u>
If the data points show that the function behaves like an exponential function, we can build a model using the exponential regression feature in the TI-83/84 calculator.

<u>Exponential Regression</u>
A computational technique that produces an exponential model that best fits the data set. The model of best fit is determined by minimizing the SSE (sum of squared errors) and AE (average error).

<u>Model Error</u>
Since the actual data points may or may not lie on the curve of best fit, the model error for a specific data point is calculated by finding the vertical distance between the point and the curve.

$$Error = actual\ value - predicted\ value$$

The *<u>actual value</u>* is the y value of the data point for a given x and the *<u>predicted value</u>* is the y value the model gives for the same value x.

- Data points that lie above the curve will produce positive errors and the model *<u>under predicts</u>*
- Data points that lie below the curve will produce negative errors and the model *<u>over predicts</u>*
- The average error (AE) of the model is found by using the SSE (Sum of Squared Errors) where n is the total number of data points in the set

$$AE = \sqrt{\frac{SSE}{n}}$$

Finding the Curve of Best Fit in the Calculator

1. Go to STAT→Edit…
2. Enter the x values in L_1
3. Enter the y values in L_2
4. Go to STAT→CALC→ExpReg
5. Type L_1, L_2, Y_1 (Y_1 is under VARS)
6. You will see the values for a, b and the line will be stored in Y_1
7. The model is of the form $f(x) = a(b)^x$

Example 1

The following data set is the population of a city (y) measured in thousands where x is the number of years after a major flood. Find the curve of best fit for data set 1.

Data Set 1							
x	0	5	8	11	15	18	22
y	179.5	168.7	158.1	149.2	141.7	134.6	125.4

Finding the SSE in the Calculator

1. Go to STAT→Edit…
2. Enter the x values in L_1
3. Enter the y values in L_2
4. Enter $Y_1(L_1)$ in L_3
5. Enter $L_2 - L_3$ in L_4
6. Enter L_4^2 in L_5
7. Go to 2nd →STAT→MATH→sum(L_5)

Example 2

Find the SSE and AE for data set 1. Round your answers to 2 decimal places.

SSE:

AE:

Example 3

What is the correlation coefficient for data set 1? What does it say about this model?

Example 4
What is the decay factor for this model? Round your answer to three decimal places.

Example 5
By what percentage does the population decline each year? Round your answer to two decimal places.

Example 6
How long will it take the population to fall to 100 thousand? If the flood happened on January 1, 1870, when does the population fall to 100 thousand? Give the year and month.

Example 7
What is the predicted population of the city at 10, 20 and 30 years after the flood?

Example 8
What is the error for population measurements at 5 and 8 years after the flood?

Newton's Law of Cooling

When an object at temperature T_0 is placed in an environment at constant temperature T_s, where $T_s < T_0$, the object will cool until it reaches T_s and no further. We can model the temperature of the object at any given time t by

$$T(t) = PTD(t) + T_s$$

where $PDT(t)$ is the positive temperature difference and is modeled by a decreasing exponential function. T_s is the temperature of the surroundings.

Example 9

You buy a hot cup of coffee on a cold day and accidentally leave it outside. The coffee was 160°F at $t = 0$ and the temperature outside is 37°F. The following data set are the actual temperatures (y) after sitting outside for x minutes. Answer the following:

Data Set 2							
x	0	5	10	15	20	25	30
y	160	138	120	104	91	81	74

(a) Predict the temperature of the coffee at 40 min. Round your answer to two decimal places.

(b) When will the coffee cool to 40°F? Round your answer to the nearest minute.

(c) What is the SSE, AE and correlation coefficient of the model? Round your answers to two decimal places.

Newton's Law of Heating

When an object at temperature T_0 is placed in an environment at constant temperature T_s, where $T_s > T_0$, the object will heat until it reaches T_s and no further. We can model the temperature of the object at any given time t by

$$T(t) = T_s - PTD(t)$$

where $PDT(t)$ is the positive temperature difference and is modeled by an increasing exponential function. T_s is the temperature of the surroundings.

Example 10

To cook your favorite holiday turkey, you bake it in the oven at 425°F until it reaches an internal temperature of 165°F. To help you, your mom gave you the following data set from her turkey last year. x is the time in hours after the turkey was placed in the oven and y is the temperature of the turkey taken with an internal thermometer.

Data Set 3: Turkey					
x	0	1	2	3	4
y	40	105	128	155	170

(a) If you put the turkey in at 1PM, what time will it reach 165°F? Round your answer to the nearest minute.

(b) If you fall asleep and forget to take it out in time, what temperature will it be at 6PM? Round your answer to the nearest degree.

(c) When will the turkey reach 350°F? Round your answer to the nearest minute.

Example 11

Fortunately, your turkey was successful, but you realize after the fact that your mom's model was incorrect due to the difference in size of your respective turkeys. At 1PM your turkey was 40°F and by 1:30PM your turkey had heated to only 52°F.

(a) If you buy the same size turkey next year, what model should you use if you still plan bake it at 425°F?

(b) How long will it take to reach an internal temperature of 165°F? Round your answer to two decimal places.

(c) If you start the turkey at 1PM, what time will it be done? Round your answer to the nearest minute.

<u>Section 5.4 Logarithmic Functions</u>

Recall from last time…

<u>Compound Interest</u>
An account with interest rate r compounded n times a year will grow using the following equation:

$$A = A_0\left(1 + \frac{r}{n}\right)^{nt}$$

However, as n gets *<u>really</u>* big (like real big), the units of time get so small that we can model the growth of the account with the *<u>continuous</u>* compound interest formula:

$$A = A_0 e^{rt}$$

where r is the annual interest rate and t is the time in years.

Example 12
You deposit $3000 in an account with an annual rate of 5%,
(a) Find the balance of the account after 3 years. Round your answer to the nearest cent.

(b) When will your account balance be $4000? Assume no withdrawals are made. Round your answer to the nearest cent.

<u>Effective Annual Yield</u>
The effective annual yield is computed using the following:

$$EAY = e^r - 1$$

Example 13
Find the EAY for the account in example 12. What does this mean?

Continuous Growth and Decay

If a quantity continuously grows or decays, we can model it by the following:

$$A(t) = A_0 e^{rt}$$

- $r > 0$ (positive) if there is continuous growth
- $r < 0$ (negative) if there is continuous decay
- t can be measured in any unit of time
- A_0 is the initial quantity when $t = 0$

Example 14

Jen really hates spiders. Her house is infested and her exterminator tells her that the pesticide has a continuous kill rate of 0.90%.

(a) If the exterminator estimates that there are 10,000 spiders in her basement (gross), how long until half of the spiders are gone? Round your answer to two decimal places.

(b) How many spiders are in her basement 24 hours after he sprays his pesticide? Round your answer to the nearest spider.

(c) Find the hourly percent decay rate. Round your answer to two decimal places.

Inverse Functions
If $g(x)$ and $f(x)$ are functions where $g(x)$ is the inverse of $f(x)$ and vice versa, then $g(f(x)) = x = f(g(x))$.

Logarithmic Functions
Logarithmic functions are the inverse of exponential functions. The two most common are

- Common Log (base 10)
  - $\log(10^x) = x$ for all x
  - $10^{\log x} = x$ for $x > 0$
  - $\log(b) = x \to 10^x = b$
- Natural Log (base e)
  - $\ln(e^x) = x$ for all x
  - $e^{\ln x} = x$ for $x > 0$
  - $\ln(b) = x \to e^x = b$

Example 15

$$30e^{0.25t} = 120$$

(a) Solve for t. Round your answer to two decimal places.

(b) What is the growth rate and continuous growth rate? Round your answer to two decimal places.

$$100(0.90)^t = 500$$

(c) Solve for t. Round your answer to two decimal places.

(d) What is the decay and continuous decay rate? Round you answer to two decimal places.

Section 5.5 Modeling with Logarithmic Functions

How do you decide to model with an exponential or logarithmic function?

- If the data set is increasing nonlinearly, we look at the concavity to determine which model is appropriate
  - Exponential growth functions are concave up
  - Logarithmic growth functions are concave down
- If the data set is decreasing nonlinearly, both exponential AND logarithmic decay functions are concave up. Technically both methods would work. However, the best model can be determined by the following:
  - Comparing their respective SSE values. The best model will have the smallest SSE
  - Comparing their respective AE values. The best model will have the smallest AE
  - Compare their respective r^2 values. The best model will have the largest r^2.

Example 16

Which is the best model for the following data set?

t	2	12	22	32
A	186.71	98.06	69.06	56.23

Exponential Model?

Logarithmic Model?

Which one is better? Why?

Useful Formulas

$$error = actual - predicted$$

$$AE = \sqrt{\frac{SSE}{n}}$$

$$A(t) = A_0\left(\frac{A_N}{A_0}\right)^{\frac{t}{N}}$$

$$A(t) = A_0 e^{rt}$$

$$EAY = e^r - 1$$

Newton's Law of Cooling

$$T(t) = PDT(t) + T_s$$

Newton's Law of Heating

$$T(t) = T_s - PDT(t)$$

$$PDT(t) = ExpReg\ model$$

$$\log(10^x) = x$$

$$10^{\log x} = x$$

$$\ln(e^x) = x$$

$$e^{\ln x} = x$$

Additional Practice Problems

1. The following table shows Makebelievia's Vibranium production for the years between 2000 and 2005. Let t=0 represent the year 2000.

Year	2000	2001	2002	2003	2004	2005
Vibranium	39	63	101	144	199	280

   (a) Find the linear model that best fits this data
   (b) Find the exponential model that best fits this data. What is the initial value, what is the growth factor, what is the growth rate?
   (c) Which of these models is a better choice? Use it for the rest of these questions.
   (d) The model [overpredicts — under predicts — exactly predicts] 2002's vibranium production.
   (e) What is the SSE, average error and correlation coefficient?
   (f) When does the vibranium production exceed 1000?
   (g) Predict the vibranium production in 2015.

2. A mathematician cooks a cake in a 350 degree oven. The table below shows the internal temperature of the cake.

Time (minutes)	0	5	10	15	20	25
"Cake" Temperature (F)	67.7	114.8	154.1	186.6	214.9	236.1

   (a) Find the Newton's Law of Heating equation for this data.
   (b) Find the SSE, average error, and correlation coefficient for this model.
   (c) Predict the cake's temperature when the mathematician pulls it out at 45 minutes. (Extra credit: Predict the look on the mathematician's face when he pulls this "cake" out)
   (d) What is the exact time this cake passes 212 degrees F.

3. Makebelievia Credit Union offers a savings account with 3.8% interest compounded monthly. Pietro opens an account with $2123.
   (a) How much is the account worth in 6 months?
   (b) When does the account reach $3000?
   (c) What is the expected annual yield?

4. Makebelievia Credit Union also offers a Certificate of Deposit with 7.3% interest compounded continuously. Wanda opens an account with $6382.
   (a) How much is the account worth in 2 years?
   (b) When does the account reach $10,000?
   (c) What is the expected annual yield?

5. Which of these accounts has the best return on investment?

Interest Rate	8.72%	8.32%	8.07%	7.94%	7.61%	7.50%
Compounded	annually	quarterly	monthly	weekly	daily	continuously

6. Palladium-103 has a half-life of 17 days, The reactor that powers the capital of Makebelievia has 23.1 kg of Palladium on January 1, 2016.
   (a) How much palladium is there on March 1st (60 days later)?
   (b) When does the amount of palladium fall below 7 kg?
   (c) What is the daily decay rate?
   (d) What is the continuous daily decay rate?
   (e) What is the monthly (30 days) decay rate?

7. Does the following data match a linear function, exponential function, logarithmic function, or none of these?

x	0	1	2	3	4	5
y	7.100	3.950	.800	-2.350	-5.500	-8.650

8. Does the following data match a linear function, exponential function, logarithmic function, or none of these?

x	0	1	2	3	4	5
y	3.100	3.720	4.464	5.357	6.428	7.7137

9. Does the following data match a linear function, exponential function, logarithmic function, or none of these?

x	1	2	3	4	5	6
y	2.460	3.257	3.723	4.054	4.311	4.521

Answers:

1. (a) $V(t) = 47.3143t + 19.3809$
   (b) $V(t) = 42.273 * 1.47746^t$, initial value = 42.273, growth factor = 1.47746, growth rate = 47.746%
   (c) Exponential (r^2 is closer to 1)
   (d) Under predicts
   (e) SSE = 461.66, average error = 9.61, correlation coefficient = .9959
   (f) $t = 8.11$ so 2009
   (g) 14750.10
2. (a) $D(t) = 282.12 * .9641^t$
   (b) Find the SSE = 1.2307, average error = .4961, correlation coefficient = −.9999
   (c) D = 54.65, T = 295.35. (Extra credit: :'( )
   (d) t = 19.61
3. (a) $2163.66
   (b) t = 9.11, 9 years and 3 months
   (c) 3.867%
4. (a) $7385.22
   (b) t = 6.15
   (c) 7.573%
5. E.A.Y.'s:

Interest Rate	8.72%	8.32%	8.07%	7.94%	7.61%	7.50%
Compounded	annually	quarterly	monthly	weekly	daily	continuously
E.A.Y.	8.72%	8.58%	8.37%	8.26%	7.91%	7.79%

   Annual is best.

6. (a) 2.0 kg
   (b) t = 29.28
   (c) r = −3.995%
   (d) r = 4.077%
   (e) r = −79.544
7. linear
8. exponential
9. logarithmic