

MATH 3200 Introduction to Higher Mathematics

Text: There are various commercial texts available under titles like "Introduction to Advanced Mathematics". C. Penney has also developed a "bare-bones" set of notes for the course.

Syllabus:

1. Logic (3 weeks)  
This should neither be rushed nor made over-technical. Students need a working knowledge of the meaning of implication and the role of universal and existential quantifiers. While some formal symbolization may be useful in getting this across, the main emphasis should be on applying these concepts smoothly and grammatically in a variety of mathematical contexts. For example, there should be exercises requiring students to negate statements from high school geometry, calculus, and linear algebra.
2. Set Theory (2 weeks)  
This provides students with their first opportunity to write proofs in a setting where the relevant axioms and definitions are explicit. This is a particularly good time to have students put problems on the board. It's sometimes useful to have several students write up the same problem and compare different styles.
3. Functions (3 weeks)  
You've done a good job if you can get your students to understand and prove statements like "composites of injective functions are injective".
4. Relations (3 weeks)  
Equivalence relations are a must. Some texts also treat binary relations and order relations, both of which are nice preparation for later courses.
5. Integers (2 weeks)  
Besides covering standard mathematical induction, there is a lot of leeway here. You may discuss the well-ordering property or treat  $\mathbb{Z}$  axiomatically.

You may decide to cover these topics in some other order, but this material should be the core of every MATH 3200 course. There should be time for one or two "enrichment topics" like cardinality. The choice is not important, but try to keep some balance between different areas of mathematics.

Classroom Technique:

The emphasis at this stage should be on logical structure and style, leaving the more creative aspects of proof-construction to other courses. Above all, students need criticism of their work: homework assignments should be collected frequently and graded carefully, and students should be encouraged to criticize each other's boardwork.

Just before advising begins, some class time should be spent reviewing major/minor requirements, possible sequels to 3200, etc.