

COPY OF
SYLLABUS

MATH 8310: SHEAVES, SCHEMES AND RIEMANN-ROCH

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General reference: [Har77]

1. SHEAVES

- (1) Sheaves of sections. Vector bundles.
- (2) General sheaves. Stalks.
- (3) Morphisms. Mono- and epimorphisms. Exact sequences.
- (4) Basic operations, including f_* , f^{-1} , $f!$, \varinjlim , \varprojlim .
- (5) Structure sheaf and sheaves of modules; f^* , \otimes .
- (6) Locally free, invertible, flasque, flat etc. sheaves.
- (7) Schemes and analytic spaces as spaces with structure sheaves.

2. SHEAVES ON ALGEBRAIC VARIETIES (AND SCHEMES)

- (1) Sheaves $\mathcal{O}(n)$ on \mathbb{P}^m and their cohomology.
- (2) Locally free sheaves on \mathbb{P}^1 .
- (3) Invertible sheaves and divisors, Picard group.
- (4) Jacobians of curves.
- (5) Coherent and quasicoherent sheaves.
- (6) Sheaves of ideals.

3. MORE ON ALGEBRAIC VARIETIES AND SCHEMES

- (1) Projective schemes and graded rings.
- (2) Proj of a sheaf of graded rings.
- (3) Projective bundles.
- (4) Blowups.
- (5) Regular imbeddings / locally complete intersections.

4. COHOMOLOGIES OF SHEAVES

- (1) Existence and basic properties: quick review/reminder.
- (2) Quasicoherent sheaves on affine varieties/schemes are acyclic.
- (3) Basic vanishing theorems for quasicoherent sheaves: $H^i(X, F) = 0$ for $i > \dim X$, Serre's vanishing.
- (4) More vanishing theorems: Kodaira, Kawamata-Viehweg.

5. INTERSECTION THEORY, CHOW RING, CHERN CLASSES

As much as necessary for the final section. Additional reference: [Ful98]

Date: Aug 18, 2004.

6. GROTHENDIECK-HIRZEBRUCH-RIEMANN-ROCH

Additional references: [Hir95, FL85]

REFERENCES

- [FL85] William Fulton and Serge Lang, *Riemann-Roch algebra*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 277, Springer-Verlag, New York, 1985.
- [Ful98] William Fulton, *Intersection theory*, second ed., Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], vol. 2, Springer-Verlag, Berlin, 1998.
- [Har77] Robin Hartshorne, *Algebraic geometry*, Springer-Verlag, New York, 1977, Graduate Texts in Mathematics, No. 52.
- [Hir95] Friedrich Hirzebruch, *Topological methods in algebraic geometry*, Classics in Mathematics, Springer-Verlag, Berlin, 1995, Translated from the German and Appendix One by R. L. E. Schwarzenberger, With a preface to the third English edition by the author and Schwarzenberger, Appendix Two by A. Borel, Reprint of the 1978 edition.