3. Write the negation of each statement in Exercise 2.
4. Write the negation of each of the following statements.
   (a) All triangles are isosceles.
   (b) Some even numbers are multiples of three.
   (c) Every door in the building was locked.
   (d) All new cars have something wrong with them.
   (e) Some angles of a triangle are greater than 90 degrees.
   (f) There are sets that contain infinitely many elements.
5. Write the negation of each of the following statements.
   (a) There is a real number x such that $x^2 + x + 1 = 0$.
   (b) Every real number is less than 100.
   (c) If f is a polynomial function, then f is continuous at 0.
   (d) If f is a polynomial function, then f is continuous everywhere.
   (e) $\forall\, x$, x real, $\exists$ a real number $y \ni y = x^3$.
   (f) There is a real-valued function $f(x)$ such that $f(x)$ is not continuous at any real number x.
6. Consider the following statement P: "The square of an even integer is divisible by 4."
   (a) Write P as a statement in the form, "for all . . . , if . . . , then . . . ."
   (b) Write the negation of P.
   (c) Prove P or $\neg P$. Explain any inductive reasoning you use to conjecture that P is true or that P is false.
7. Consider the following statement P: "The sum of two even integers is divisible by 4."
   (a) Write P as a statement in the form "for all . . . , if . . . , then . . . ."
   (b) Write the negation of P.
   (c) Prove P or $\neg P$. Explain any inductive reasoning you use to conjecture that P is true or that P is false.
8. In Example 14 the definition of a bounded function was given.
   (a) Write the negation of this definition; that is, complete the following statement: "A real-valued function $f(x)$ is *not bounded* on the closed interval $[a, b]$ if . . . ."
   (b) Give an example of a bounded function on $[0, 1]$. Justify your answer by determining a value for M.
   (c) Give an example of an unbounded function on $[0, 1]$. Justify your answer.
   (d) Suppose the definition of bounded function were worded this way: "A real-valued function $f(x)$ is said to be *bounded* on the closed interval $[a, b]$ if for all $x \in [a, b]$, there exists a positive real number M such that $|f(x)| \leq M$." Does this definition mean the same as the one given in Example 14? If not, explain how they differ. Could this new definition make sense as the definition of a bounded function? Explain.

9. A real-valued function $f(x)$ is said to be *increasing* on the closed interval $[a, b]$ if for all $x_1, x_2 \in [a, b]$, if $x_1 < x_2$, then $f(x_1) < f(x_2)$.
   (a) Write the negation of this definition.
   (b) Give an example of an increasing function on $[0, 1]$.
   (c) Give an example of a function that is not increasing on $[0, 1]$.
10. (a) State a definition for a real-valued function $f(x)$ to be *decreasing* on a closed interval $[a, b]$.
   (b) Give the negation of this definition.
   (c) Give an example of a decreasing function on $[0, 1]$.
   (d) Give an example of a function on $[0, 1]$ that is neither increasing nor decreasing.
11. Prove the following corollary of the Archimedean Principle. (See Example 28 for the statement.) For every positive real number ε, there exists a positive integer N such that $1/n < \varepsilon$ for all $n \geq N$. (Note: this exercise is the basis for the formal proof that the sequence $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$ converges to 0.)
12. Use the Archimedean Principle to prove the following: if x is a real number, then there exists a positive integer n such that $-n < x < n$.
13. Prove that if x is a positive real number, then there exists a positive integer n such that $\frac{1}{n} < x < n$.

Discussion and Discovery Exercises

D1. Consider the following question. What positive integers n can be written as the difference of two squares? For example, $5 = 3^2 - 2^2$ and $24 = 5^2 - 1^2$. The following table lists the expression $n = x^2 - y^2$ for varying values of x and y. Since n is positive, we assume that $x > y \geq 0$.

x	y	$x^2 - y^2$	x	y	$x^2 - y^2$	x	y	$x^2 - y^2$
1	0	1	5	4	9	8	0	64
2	0	4	6	0	36	8	1	63
2	1	3	6	1	35	8	2	60
3	0	9	6	2	32	8	3	55
3	1	8	6	3	27	8	4	48
3	2	5	6	4	20	8	5	39
4	0	16	6	5	11	8	6	28
4	1	15	7	0	49	8	7	15
4	2	12	7	1	48	9	0	81
4	3	7	7	2	45	9	1	80
5	0	25	7	3	40	9	2	77
5	1	24	7	4	33	9	3	72
5	2	21	7	5	24	9	4	65
5	3	16	7	6	13	9	5	56