

MATH4900
Topics in Mathematics
Transforms

Pre-requisites: MATH3000

Expected outcomes: Understanding of the most common linear transforms (Fourier, SVD, Radon and some others), their properties and their applications.

Course Description: The basic outline for this course is to discuss four different classes of transforms: 1) (Basics) We will start off with a review of eigenvalue decompositions and the spectral theorem. 2) (Fourier) From there, we'll discuss graphs and adjacency matrices then introduce the discrete Fourier transform, which will be introduced as the eigenvalue decomposition of circulant matrices. Basics of Fourier transforms, such as the Plancherel theorem, convolution, etc. We will discuss applications such as the solution of the discrete heat equation and properties such as translation and dilation. Next, the Fourier transform will be discussed on the real numbers with application to the heat equation. Finally, the discrete Fourier transform will be re-introduced as a transform on a finite abelian group. This will allow a natural transition to discussion of the Hadamard transform with the application to error correcting codes. 3) (Singular value decomposition (SVD, discrete) and Karhunen-Loeve transform (continuous)) This section will begin with a derivation of the properties of the SVD as a generalization of the spectral decomposition. The SVD will then be rederived as a decomposition providing the best possible separable approximation of a matrix. Applications to multivariate data analysis will be discussed. Finally, the relationship of the SVD with the multivariate normal distribution and its principal axes will be discussed. 4) (Tomography and Radon transform) The final transforms to be discussed will be general discrete tomography transforms and the Radon transform. Applications to x-ray tomography and remote sensing will be discussed.

I have not yet determined a single text, a selection of a few parts of texts may be used. Section 1 is covered in 'Linear Algebra, a Geometric Approach' by Shifrin and Adams. Sections 1 and 2 are covered in 'Fourier Analysis on Finite Groups and Application' by Audrey Terras. A good discussion of the SVD and Karhunen-Loeve transform material may be found in 'Approximation of Large-Scale Dynamical Systems' by Athanasios Antoulas. I don't yet know a good reference for the Radon transform and tomography material.

Honor Code: *As a University of Georgia student, you have agreed to abide by the University's academic honesty policy, "A Culture of Honesty," and the Student Honor Code. All academic work must meet the standards described in "A Culture of Honesty" found at: www.uga.edu/honesty. Lack of knowledge of the academic honesty policy is not a reasonable explanation for a violation. Questions related to course assignments and the academic honesty policy should be directed to the instructor.*

As applies to this course, this means do your own homework (you can work together, but what you write should be your own) and exams and projects should be your own work.

The course syllabus is a general plan for the course; deviations announced to the class by the instructor may be necessary.