

1. (24 points) Evaluate those of the following limits which exist and write “does not exist” for the others.

a)  $\lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x^2 - 4x + 3}$

b)  $\lim_{x \rightarrow 5} \frac{x^2 - 7x + 10}{x^2 - 3x - 10}$

c)  $\lim_{x \rightarrow 3^+} \frac{|3 - x|}{x - 3}$

d)  $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$

2. (24 points)

a) Complete the definition : The *derivative* of a function  $f$ , denoted by  $f'(x)$ , is

b) Use the definition to find  $f'(x)$  for the function  $f(x) = \sqrt{2x + 1}$ .

c) Find the equation of the line tangent to the curve  $y = \sqrt{2x + 1}$  at the point  $(4, 3)$ .

**3.** (24 points) A ball is thrown upwards from an initial height of 24 ft. Its height  $y$  in feet  $t$  seconds later is given by the formula

$$y = f(t) = -16t^2 + 40t + 24.$$

- a) When is the (instantaneous) velocity of the ball 16 feet per second?
- b) What is the ball's maximum height?
- c) When does the ball hit the ground?
- d) What is the average velocity of the ball over the time interval during which it is airborne?

(space for work)

4. (6 points) Complete the definition : The function  $f$  is *continuous* at the number  $a$  if

5. (22 points) Take  $f(x) = \frac{x^2 - 2x - 3}{x^2 - 4x + 3}$  for all parts of this problem. You must include appropriate prose to get full credit for any part.

a) Find all *removable* discontinuities of  $f$ .

b) Find all asymptotes of the graph of the curve  $y = f(x)$ .

c) Use the Intermediate Value Theorem to solve the inequality  $f(x) \geq 0$ .

**Bonus.** (10 points) Find equations of all lines passing through the point  $(2, 3)$  which are tangent to the parabola  $y = x^2$ . Note: The point  $(2, 3)$  does not lie on the parabola.