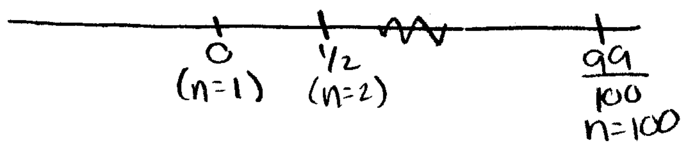
REVIEW for 1st test

$S=\left\{\frac{n-1}{n} : n\in\mathbb{N}\right\}$

Find LUB & GLB of S



LUB of S = 1
GLB of S = 0

Prove that 1 really is the LUB

(1) show 1 is an upper bound of S

Let $x\in S$

Then $x=\frac{n-1}{n}$ for some $n\in\mathbb{N}$

$n-1<n$ so

$x=\frac{n-1}{n}<1$

$\therefore$ 1 is indeed an upper bound of S

(2) suppose $c<1$

$\therefore$ c is not an upper bound of S

This means 1 is the LUB of S

$\therefore 1-c>0$

by the arch Find some $n\in\mathbb{N}$ s.t. $n(1-c)>1$

$\therefore 1-c>\frac{1}{n}$

So $\frac{n-1}{n}=1-\frac{1}{n}>c$

$\mathbb{N} \to \mathbb{R}$ $S = \{x \in \mathbb{R} \mid x > 0 \text{ \& } x^2 \leq 2\}$

LUB $S = \sqrt{2}$

$\mathbb{N}$ $\mathbb{R}$ $S = \{x \in \mathbb{R} \mid x > 0 \text{ \& } x^2 < 2\}$

LUB $S = \sqrt{2}$

$\mathbb{N}$ $\mathbb{Q}$ $S = \{x \in \mathbb{Q} \mid x > 0 \text{ \& } x^2 < 2\}$

no LUB in $\mathbb{Q}$

$S = \{x \in \mathbb{Z} \mid x > 0 \text{ \& } x^2 < 2\} = \{1\}$

LUB & GLB $= 1$

$$
\begin{aligned}
(a+b)(a-b) &= (a+b)(a+(-b)) \\
&= (a+b)(a) + (a+b)(-b) \\
&\overset{\text{comm.}}{=} a(a+b) + (-b)(a+b) \\
&\overset{\text{dist}}{=} a^2 + ab + (-b)(a) + (-b)(b) \\
&=
\end{aligned}
$$

prop 1.e — F is an ordered field

If $x \neq 0$, then $x^2 \in F^+$

apply trichotomy to x:

Case 1: $x \in F^+$. by closure under multiplication, $x \cdot x \in$ [illegible]

$x \cdot x = x^2$

Case 2: $-x \in F^+$. by closure under mut. $(-x)(-x) \in F^+$

$(-x)(-x) = x^2$ so $x^2 \in$ [illegible] prop

If $x \in F^+$, then $x^{-1} \in F^+$

we apply trichotomy to x^{-1}

case 1: $x^{-1} = 0$

then $1 = \boxed{x(x^{-1}) = x(0)} = 0$

impossible

case 2: $-x^{-1} \in F^+$

then $\underbrace{(x)(-x^{-1})}_{-1} \in F^+$

but $1 = 1^2 \in F^+$

since we know $1 \in F^+$, by trichotomy, -1 cannot be in

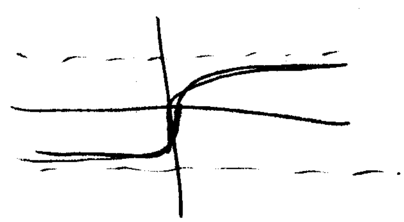
impossible

since ① + ② do not work, this implies that x^{-1} has to be in F^+ by trichotomy

---

#39 $y = \arctan$ is an example of a strictly increasing function which is eventually positive, but does not diverge to ∞

& $\frac{x}{1+|x|}$



#38 $f(x) = x^2 + ax + b$ is eventually positive

proof: $f(x) = x(x+a) + b$

[want $x + a > 1 \Rightarrow x > (1-a)$
$x > -b$]

Take $N = \max\{1-a, -b, 0\}$

suppose $x > N$

<u>case 1:</u> $b \geq 0$

$x > 0$ & $x > 1-a \Rightarrow (x+a) > 1$

$f(x) = x(x+a) + b > 0$.

<u>case 2:</u> $b < 0$

then $-b > 0$

$\therefore$ $x(x+a) > (-b)(1) = -b$

$f(x) = x(x+a) + b > 0$

$f(x) = x^2 - 3x - 2 = x(x-3) - 2$ diverges to ∞

<u>proof</u>. Let $M > 0$ be given

want:

$x^2 - 3x - 2 - M > 0$

$x(x-3) - (2+M) > 0$

It is enough to make $x > 2+M$ & $x - 3 > 1 \Rightarrow x > 4$

Take $N = \max(2+M, 4)$

Suppose $x > N$, then $x > 2+M$ & $x > 4$

$(x-3) > 1$

therefore, $f(x) - M = x(x-3) - (2+M) > (2+M)(1) - (2+M) = 0$

#10 Δ ineq. $|x+y| \leq |x| + |y|$

$|a| = |(a-b) + (b)| = |a-b| + |b|$

$\therefore$ $\underline{|a| - |b|} \leq |a-b|$

switch $a + b$, then $\underline{|b| - |a|} \leq |b-a| = |(-1)(a-b)| =$

#30 if $|x-\frac{1}{10}|<\frac{1}{20}$ then $|x|>\frac{1}{20}$

proof: $\boxed{-\frac{1}{20}<x-\frac{1}{10}}<\frac{1}{20}$

$\frac{1}{10}-\frac{1}{20}<x$

$\frac{1}{20}<x \quad \therefore \ |x|=x>\frac{1}{20}$ ∎

#29 suppose $|x-5|<1$

(b) $|x^2+3x-40| = |x+8||x-5| < (14)(1) = 14$

(a) $|x+8|<14$

pf: $|x+8| = |(x-5)+(13)| \leq |x-5|+13 < 1+13 = 14$

another way

$-1<x-5<1$

$4<x<6$

$12<x+8<14$

$|x+8| = x+8<14$

---

newton's method know!

$x_1 = 2$

for $n>1$, $x_{n+1} = x_n^2+5 \ (*)$

Find x_3

take $n=1$ in $(*)$, then $x_2 = x_1^2+5 = 4+5 = 9$

" $n=2$ " $x_3 = x_2^2+5 = 81+5 = 86$

use newton's method to approximate $\sqrt[3]{5}$

want $x=\sqrt[3]{5}$
ie $x^3=5$
ie $x^3-5=0$

apply newton's method w/ $f(x)=x^3-5$
$f'(x)=3x^2$

take $x_1=1$

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

monic poly: leading coefficient is 1.

$$x_{n+1}=x_n-\frac{x_n^3-5}{3x_n^2}$$

$$x_{n+1}=\frac{2}{3}x_n+\frac{5}{3x_m^2}$$

taking $n=1$, we get

$$x_2=\frac{2}{3}x_1+\frac{5}{3x_1^2}=\frac{2}{3}+\frac{5}{3}=\frac{7}{3}$$

$$x_2=x_1-\frac{x_1^3-5}{3x_1^2}=1-\frac{(1-5)}{3}=1-\left(\frac{-4}{3}\right)=\frac{7}{3}$$

#11 $x^2=6x$
div. by x & get $x=6$ is the only soln
(not legit to divide by x when $x=0$

#57 $S=\{n\in\mathbb{N}\mid$ every monic polynomial of degree n diverges to ∞

pf suppose $p(x)$ is a monic poly of deg. 1
$p(x)=x+b$
suppose M is given [want: $x+b>M$ when $x>M-b$]

Take $N = M - b$

suppose $x > N$

$x > M - b$

$p(x) = x + b > M$

$\therefore 1 \in S$

Suppose $k \in S$

suppose p is a monic polynomial of degree $k+1$

$p(x) = x^{k+1} + a_k x^k + a_{k-1} x^{k-1} + \dots + a_1 x + a_0$

$= x\underbrace{(x^2 + a_k x^{k-1} \pm \dots \pm a_1)}_{q(x)} + a_0$

suppose $M > a_0$ be given

scratch
want
$x \cdot q(x) + a_0 - M > 0$