

## SAMPLE FIRST HOUR EXAM (100 POINTS)

Points will be deducted for imprecise or disorganized notation.

1 (20 points). Compute the following limits. If a limit does not exist, explain why.

- a)  $\lim_{x \rightarrow 5} \frac{x^2 - 25}{3x + 1}$   
 b)  $\lim_{x \rightarrow 3^-} \frac{|x - 3|}{x^2 - 9}$   
 c)  $\lim_{x \rightarrow 0} \frac{\sin(5x^2)}{3x}$   
 d)  $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{x^2 - 6x + 5}$

2 (25 points). Let  $f(x)$  be a function.

- a) Express the definition of the derivative  $f'(x)$  in the form  $\lim_{h \rightarrow 0} \dots$   
 b) Express the definition of the derivative  $f'(x)$  in the form  $\lim_{t \rightarrow x} \dots$   
 c) Use **one of the definitions above** to compute  $f'(x)$  if  $f(x) = \frac{1}{4x}$ .  
 d) Find an equation of the line tangent to the curve of Part c) at  $x = 2$ .

3 (20 points). The height  $y$  (in feet) of a ball  $t$  seconds after it is thrown up in the air is given by the formula  $y = 80t - 16t^2$ .

- a) What is the ball's average velocity during its first two seconds of flight ?  
 b) What is the instantaneous velocity of the ball at  $t = 2$  ?  
 c) When is the ball 24 feet high ?  
 d) What is the maximum height achieved by the ball ?

4 (25 points). Give an explicit function [like  $f(x) = 3x$ ] that exemplifies each phenomenon. No explanation is required.

- a) a function that is **not a polynomial**, but is continuous on its domain  
 b) a function having a removable discontinuity  
 c) a function whose graph has the line  $y = \frac{5}{2}$  as a horizontal asymptote  
 d) a function whose graph has two vertical asymptotes  
 e) a function whose graph has a slant asymptote.

5 (10 points). State the Intermediate Value Theorem and use it to show that the equation  $2^x = 3x^2$  has a solution in the interval  $[-1, 0]$ . You must include appropriate prose for full credit.