September 24, 2009

<u>Review for Test #1</u>

$\|c\vec{x}\| = |c|\,\|\vec{x}\|$

$\vec{x} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ $\|x\| = \sqrt{5^2 + 8^2} = \sqrt{89}$

$\frac{1}{\sqrt{89}} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 5/89 \\ 8/89 \end{bmatrix}$ $\|\vec{y}\| = \sqrt{(5/\sqrt{89})^2 + (8/89)^2}$

Given $\begin{cases} \vec{v} \text{ (on line } l) \\ \vec{y} \end{cases}$ {Find the distance between a point and a line}

$\vec{y} = \underbrace{\vec{y}_{\parallel}}_{\text{proj of } y \text{ on } l} + \underbrace{y_{\perp}}_{\text{proj of } y \text{ orthogonal to } l}$ where $\begin{cases} \vec{y}_{\parallel} \parallel \vec{v} \\ \vec{y}_{\perp} \perp \vec{v} \end{cases}$

*orthogonal in front of projection does not mean anything.

<u>Section 1.3 in Text</u> : Hyperplane in $\mathbb{R}^n$

Given $\vec{v} \in \mathbb{R}^n$ $v \neq \vec{0}$ $\vec{v} = (v_1, \dots, v_n)$

$b \in \mathbb{R}$

The corresponding hyperplane is $\{\vec{x} \in \mathbb{R}^n \mid \vec{v} \cdot \vec{x} = b\}$

$\{\vec{x} \in \mathbb{R}^n \mid v_1x_1 + \dots + v_nx_n = b\}$

examples ① $n = 2$ $b = 0$ $\vec{v} = (1, 2)$

$H = \{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbb{R}^2 \; x_1 + 2x_2 = 0\}$

(line through the origin)

② $n = 2$ $b = 5$ $\vec{v} = (1, 2)$

$H = \{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbb{R}^2 \mid x_1 + 2x_2 = 5\}$

line $\parallel$ ① 1-Dimension

③ $n = 3$ $b = 0$ $v = (1, 2, 3)$

$H = \{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbb{R}^n \mid x_1 + 2x_2 + 3x_3 = 0\}$ In $\mathbb{R}^3$ = 2D or 1D

Plane (2-Dimension) in $\mathbb{R}^3$

Hyperplane is a solution to a single equation. It's one dimension less THAN THE AMBIENT SPACE $\mathbb{R}^n$.

9/24/09

Descriptions of a space
(1) Parametric
(2) Cartesian = Constraint = Intersections of Hyperplanes.

∘ Defn: The rank of a matrix is the number of pivots in its echelon form. Classify solution types of
$A_{m \times n}\vec{x} = \vec{b}$ $(A, \vec{b})$ Fixed
1) Unique $\Leftrightarrow$ Rank A = Rank$[A \mid b] = m$
2) No solutions $\Leftrightarrow$ Rank $A <$ Rank $[A \mid b]$
3) ∞-ly many solutions $\Leftrightarrow$ Rank A = Rank $[A \mid b] < n$
The Rank of the augmented matrix EITHER EQUALS RANK A OR EQUALS $1 +$ Rank A.

$A_{m \times n}$ Given
<u>Characters of Solutions to $A\vec{x} = \vec{b}$ for different $\vec{b}$'s</u>
1) Unique solution for every $\vec{b}$. THIS HAPPENS IF AND ONLY IF
$A \to I$ (A REDUCES TO I)
$\Leftrightarrow$ Pivot in every row and a pivot in every column OF A.
$\Leftrightarrow R = m = n$.

* Only time you can always have a unique solution is when Matrix is square.

2) $R = m < n$ $A\vec{x} = \vec{b}$ always have infinetly many solutions.
3) $R = n < m$ unique for some $\vec{b}$ and no solution for other $\vec{b}$'s.

$\vec{x} = \vec{a} + t\vec{v}$
$\{\vec{a} + t\vec{v} \mid t \in \mathbb{R}\}$

$$\begin{array}{ccc} (1,1) & & \text{IF free variable.} \\ \end{array}$$
$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 5 & 3 \end{bmatrix}$$
$(2,2)$

Rank = # pivots in echelon form.
Free variables is columns that don't have pivots.

• Linear Combinations: The sum of scalar multiples of ~~a~~ SEVERAL vectors.
• Angle between vectors $\theta = \cos^{-1}\left(\frac{u \cdot v}{|v||u|}\right)$
• Echelon: The leading non zeros in later rows are to the right of the rows previous.
• Singular, non-singular refer to square matrices.
• Matrix is its own transpose $\Leftrightarrow$ Symmetric