Cohomology of finite groups of Lie type

Let G be a reductive algebraic group over a field k of prime characteristic p which is split over the prime field $\mathbb{F}_p$. Let $\mathrm{Fr}: G \to G$ denote the Frobenius map. Then the fixed points of the rth iterate of the Frobenius map, denoted $G(\mathbb{F}_{p^r})$, is a finite Chevalley group. The question of interest in this talk is to determine the least $i > 0$ such that the cohomology group $\mathrm{H}^i(G(\mathbb{F}_{p^r}), k) \neq 0$.

Quillen showed that $\mathrm{H}^i(GL_n(\mathbb{F}_{p^r}), k) = 0$ for all $0 < i < r(p-1)$ for all n. In that work, he noted that for all finite Chevalley groups there exist constants C depending on the root system such that $\mathrm{H}^i(G(\mathbb{F}_{p^r}), k) = 0$ for $0 < i < Cr$. However no explict value of C is given except for $G = SL_2$ (and p odd) in which case one can take $C = (p-1)/2$. Further, it was not shown whether these vanishing ranges were sharp. Indeed, in the case of SL_2, one can see that these bounds are not sharp in general. Later work by Friedlander and by Hiller extended Quillen's results and found vanishing ranges for groups of all types. Since then, few if any results have been obtained in this direction.

The goal here is to exploit techniques developed by Bendel, Nakano and the presenter which relate $\mathrm{H}^i(G(\mathbb{F}_{p^r}), k)$ to extensions over G. In particular, for a group G of classical type and $r = 1$, under the assumption that $p > h$ (the Coxeter number), we improve on the vanishing ranges of Hiller and in many cases find sharp bounds.