UGA Algebra Seminar — Jan. 14, 2008

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Title: Positivity in equivariant K-theory: conjectures and results

Let G be a complex semisimple group and let $B \supset T$ be a Borel subgroup and a maximal torus, respectively. Let $X = G/P$, where $P \supset B$; this is a partial flag variety. The T-equivariant K-theory of X, denoted $K_T(X)$, is a free module over the representation ring $R(T)$. One important basis consists of the classes of structure sheaves of Schubert varieties; another important basis, which has been used by Kostant and Kumar, is the dual basis to this basis. (These bases are essentially different, in contrast to the situation in equivariant cohomology where the dual basis to a Schubert basis is a Schubert basis for the opposite Borel subgroup.) We conjecture that the structure constants for the multiplication in $K_T(X)$, in the dual basis to structure sheaves, satisfy a certain "positivity" property (more precisely, they can be expressed as certain products with known signs). This property is similar to the positivity conjectured by Griffeth and Ram for the structure constants obtained using the basis of structure sheaves. As evidence for our conjecture we give an explicit formula for these constants for X equal to projective space, which implies positivity. We can also prove the conjecture in some other special cases.