MATH 2200 (Boe) SAMPLE FINAL EXAM

You must show your work to receive full credit. Use proper mathematical sentences (equals signs, parentheses, etc.). Calculators are permitted, except those which can do symbolic differentiation, such as the TI-89 and TI-92. Total points: 100.

1. (8 points) Using only the **definition** of the derivative, compute the derivative of $f(x) = \sqrt{x+4}$. (No points for using "differentiation formulas," though of course you should check your answer that way.)

2. (12 points) Compute the limits. Use the symbols ∞ and $-\infty$ whenever possible. Graphical or numerical-approximation methods are not acceptable. **Show your work.**

a) $\lim_{x\to 2} \dfrac{x^2+2x-8}{x^2-x-2}$

b) $\lim_{x\to 3^+} \dfrac{x-5}{x^2-4x+3}$

c) $\lim_{\theta\to 0} \dfrac{1-\cos\theta}{\theta^2}$

d) $\lim_{x\to -\infty} \dfrac{1-3x+2x^2}{x^3+4x^2+5}$

3. (12 points) Compute the following derivatives and integral using any method/rules.

a) $\dfrac{d}{dx}\, e^{4x}\tan(x^3+2)$

b) $\dfrac{d}{dx}\, \sin^3(\ln(x))$

c) $\displaystyle\int \left(4\sqrt[3]{x} - \frac{2}{\sqrt{x}} + \frac{3}{x}\right) dx$

d) Find $\dfrac{dy}{dx}$ if $y + e^y = x^2 y - 1$.

4. (10 points) Find the absolute maximum and minimum values, and where they occur, for the function $f(x) = \dfrac{x}{x^2+1}$ on the interval $[-3, 0]$.

5. (14 points) A mystery function $f(x)$ has first and second derivatives $f'(x) = \dfrac{3(1-x)}{x^{2/3}}$ and $f''(x) = \dfrac{-(x+2)}{x^{5/3}}$. Assume that $f(x)$ is continuous on $(-\infty, \infty)$ and that $f(0) = 0$.

(a) Find the intervals where f is increasing, and the x-coordinates of any absolute or local maxima or minima of f (specify). If f has any vertical asymptotes or vertical tangent lines, give their x-coordinates (and specify which).

(b) Find the intervals where f is concave up, and the x-coordinates of any inflection points of f.

(c) Use your answers to (a) and (b) and the other given information to sketch the graph of $y = f(x)$. [Use the back of the previous page if you need more room.]

6. (10 points) A spotlight on the ground shines on a building 30 ft away. A man 6 ft tall walks from the building toward the light at 5 ft/s. How fast is the height of his shadow on the building increasing when he is 10 ft from the spotlight? [Hint: Draw the beam of light that defines the top of his shadow.]

7. (10 points) A rectangular poster has margins of 1 inch on each side, 2 inches at the top, and 3 inches at the bottom. The area of the printed material on the poster must be 90 square inches. Find the smallest possible total area of the poster. (Be sure to **prove** you have found the smallest poster. No credit for trial-and-error, guessing, graphical solutions, etc.)

8. (8 points) A driver traveling 48 ft/s begins to apply the brakes at $t = 0$. The brakes provide a deceleration of $6t$ ft/s^2 after t seconds, until the car stops. How far does the car travel before stopping? [Hint: Deceleration is negative acceleration.]

9. (8 points) Prove that the equation $x^5 - 4x^2 + 1 = 0$ has **exactly three** different solutions. Be sure to mention any theorem(s) you use, and any crucial properties of the equation that allow you to apply those theorem(s).

10. (8 points) On July 1, 2000, the groundhog population of Athens-Clarke County was only 10, but by July 1, 2005 there were 30 groundhogs. Assuming the number of groundhogs grows continuously at a rate proportional to the population, in what year should the county's groundhog population reach 130?