

You must show your work to receive credit. Use proper mathematical sentences (= signs, parentheses, etc.). Calculators are permitted, except those which can do symbolic differentiation, such as the TI-89 and TI-92. The total number of points is 50.

1. (8 points) Use implicit differentiation to find the slope of the tangent line to the curve $xy = 6e^{2x-3y}$ at the point $(3, 2)$.
2. (8 points) Use differentials to estimate $\sqrt[3]{994}$. (You should not have to use a calculator!)
3. (12 points) Let $f(x) = x^4 - 4x^3 - 8x^2 + 3$.
 - (a) Find the open intervals where f is increasing or decreasing.
 - (b) Classify each critical point of f (as local maximum, local minimum, or neither).
 - (c) Compute $\lim_{x \rightarrow \pm\infty} f(x)$.
 - (d) Sketch the graph of $y = f(x)$, using the information you assembled in (a)–(c). [That is, your graph must be consistent with **your** answers above.]
4. (12 points) You are to design a rectangular aquarium with volume 12,000 cubic inches (about 50 gallons), with one pair of opposite ends being square. The four bottom edges and the four upright edges are to be assembled with extraordinarily expensive glue. Find the dimensions of the aquarium which minimize the total **length** to be glued. (Be sure to prove that you have found an absolute minimum. No credit for trial and error.)
5. (10 points) One bicycle is 4 miles east of an intersection, and is traveling toward the intersection at 7 mi/hr. At the same time a second bicycle is 3 miles south of the intersection and traveling away from the intersection at 11 mi/hr. Is the distance between the two bicycles increasing or decreasing at that moment, and at what rate? (Make sure your answer is reasonable!)