

Always justify your answers, even if the question does not explicitly say so! Write your own solutions, independently of anyone else.

Core Problems: Everyone must turn these problems in.

I. Sec. 1.1 # 2, 3.

II. Prove by induction (for all $n \in \mathbb{N}$):

(a) $3 + 7 + 11 + \cdots + (4n + 3) = (2n + 3)(n + 1)$.

(b)
$$\sum_{k=1}^n \frac{1}{k(k+1)} = \frac{n}{n+1}.$$

III. Sec. 1.1 # 6a (The original definition is the one at the top of p. 6. Give a combinatorial proof, as suggested in the hint.).

IV. State Rolle's Theorem from differential calculus. Using mathematical induction, prove that if f is continuous on $[a, b]$, differentiable on (a, b) , and f has n zeros on $[a, b]$, then f' has at least $n - 1$ zeroes on $[a, b]$.

V. Let n and k be positive integers. After calculating several examples, guess a closed formula (meaning no "... " or " \sum ") for

$$\binom{n}{0} + \binom{n+1}{1} + \cdots + \binom{n+k}{k};$$

it may help to stare at Pascal's triangle while computing the sums. Then prove your formula by induction.

Advanced Problem: Students registered for 6000 must turn these problems in. They count for extra credit for 4000 students, but anyone hoping to get an 'A' in 4000 should do a reasonable number of advanced problems. Advanced problems are due one week **after** the Core problems from the same assignment, unless announced otherwise. Please hand in Advanced Problems separately from Core Problems.

VI. Sec. 1.1 # 16 [Remark: This is one of the harder "Advanced Problems" I'll assign all semester. Don't be discouraged if you don't solve it!]