

Always justify your answers, even if the question does not explicitly say so! Write your own solutions, independently of anyone else.

Core Problems: Everyone must turn these problems in.

I. Sec. 1.3 # 21 ab [You must show all your work — don't merely invoke the formulas of the theorem, or use trial and error, inspection, etc.]

II. Sec. 1.3 # 22, 28, 29.

III. Prove that $n^{561} \equiv n \pmod{561}$ for all $n \in \mathbb{N}$. (Hint: $561 = 3 \times 11 \times 17$. Using Exercises 1.3.11 and 1.3.19, first prove that when $\gcd(n, 561) = 1$, $n^{560} \equiv 1 \pmod{561}$. Then use the same approach to see that when n is a multiple of 3, 11, or 17, $n^{561} \equiv n \pmod{561}$.) Remark: 561 is the smallest Carmichael number, i.e., the smallest non-prime number for which the result of Fermat's little theorem holds.

IV. Sec. 1.4 # 3, 4, 6.

Advanced Problem: Due Wed. Sep. 23. Students registered for 6000 must turn these problems in. They count for extra credit for 4000 students, but anyone hoping to get an 'A' in 4000 should do a reasonable number of advanced problems. Please hand in Advanced Problems separately from Core Problems.

V. Sec. 1.3 # 35.