UNIVERSITY OF GEORGIA
DEPARTMENT OF MATHEMATICS
2200 CALCULUS 1
Fall 2005, Kazancı

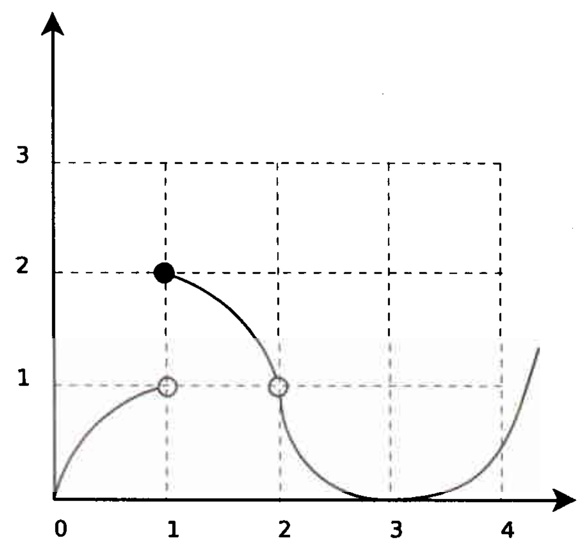
TEST 1 9/14/05

Name : SOLUTIONS

Problem	Score	Points
1		15
2		24
3		16
4		15
5		15
6		15
Total		

50 minute time limit. No books, notes or calculators are permitted. You may use the back of the page for additional space, but indicate clearly when you do so. Please show all your work on all problems. Credit will not be given without your work. Good Luck!

1. (15 pts) Graph of $f(x)$ is given below:



(a) Find the indicated one sided limit:

$\lim_{x\to 1^+} f(x)$ 2 $\qquad$ $\lim_{x\to 1^-} f(x)$ 1

$\lim_{x\to 2^+} f(x)$ 1 $\qquad$ $\lim_{x\to 2^-} f(x)$ 1

$\lim_{x\to 3^+} f(x)$ 0 $\qquad$ $\lim_{x\to 3^-} f(x)$ 0

(b) Find the indicated limit if it exists (or state that it does not exist):

$\lim_{x\to 1} f(x)$ D.N.E.

$\lim_{x\to 2} f(x)$ 1

$\lim_{x\to 3} f(x)$ 0

(c) Is $f(x)$ continuous at $x = 1$ (Why/Why not?)

No, because the limit does not exist at $x=1$.

Is $f(x)$ continuous at $x = 2$ (Why/Why not)?

No, because $f(x)$ is not defined at $x=2$.

Is $f(x)$ continuous at $x = 3$ (Why/Why not?)?

Yes, because $\lim_{x\to 3} f(x) = f(3)$

2. (24 pts) Find the indicated limit if it exists.

(a) (8 pts)

$$\lim_{x\to 1} \frac{x^2-2x+1}{x^2-3x+2}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x-1)}{\cancel{(x-1)}(x-2)} = \lim_{x\to 1} \frac{1-1}{1-2} = \frac{0}{-1} = \boxed{0}$$

(b) (8 pts)

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$$

$$= \lim_{x\to 1} \frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)} = \lim_{x\to 1} \frac{(1+x)-1}{x(\sqrt{1+x}+1)}$$

$$= \lim_{x\to 1} \frac{x}{x(\sqrt{1+x}+1)} = \lim_{x\to 1} \frac{1}{\sqrt{1+x}+1} = \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

(c) (8 pts)

$$\lim_{x\to 2} \left(\frac{1}{x-2} - \frac{2}{x^2-2x}\right)$$

$$= \lim_{x\to 2} \left(\frac{1}{x-2} - \frac{2}{x(x-2)}\right) = \lim_{x\to 2} \left(\frac{x}{x(x-2)} - \frac{2}{x(x-2)}\right)$$

$$= \lim_{x\to 2} \frac{\cancel{x-2}}{x\cancel{(x-2)}} = \lim_{x\to 2} \frac{1}{x} = \boxed{\frac{1}{2}}$$

3. (16 pts) Find the indicated limit if it exists. Give reasons why the limit does or does not exist. Find the right limit and the left limit if necessary.

(a) (8 pts)

$$\lim_{x \to 3} \frac{x+1}{x^2 - 2x - 3} = \lim_{x \to 3} \frac{x+1}{(x+1)(x-3)}$$

$$= \lim_{x \to 3} \frac{1}{x-3} = \frac{1}{0} \text{ ?}$$ Find limit from right and left.

$$\left.\begin{array}{l} \lim_{x \to 3^+} \frac{1}{x-3} = +\infty \\ \lim_{x \to 3^-} \frac{1}{x-3} = -\infty \end{array}\right\} \lim_{x \to 3} \frac{1}{x-3} \quad \text{D.N.E.}$$

(b) (8 pts)

$$\lim_{x \to 1} \frac{x-1}{2x^2 - 3x - 4}$$

$$\lim_{x \to 1} \frac{x-1}{2x^2 - 3x - 4} = \frac{1-1}{2-3-4} = \frac{0}{-5} = 0$$

4. (15 pts) Use the Intermediate Value Theorem to find an interval of length 1 which contains a solution of the following equation.

$$\underbrace{\sqrt{x-5} - \frac{1}{x-3}}_{f(x)} = 0$$

$f(x)$ is not defined when $x < 5$

$$f(5) = \sqrt{0} - \frac{1}{5-3} = 0 - \frac{1}{2} = -\frac{1}{2} < 0$$

$$f(6) = \sqrt{1} - \frac{1}{6-3} = 1 - \frac{1}{3} = \frac{2}{3} > 0$$

$f(x)$ is continuous on $[5, 6]$ and

$f(5) < 0 < f(6)$; so by I.V.T.

there exists a solution of

$$f(x) = \sqrt{x-5} - \frac{1}{x-3} = 0$$

on the interval $(5, 6)$.

5. (15 pts) Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither?

$$f(x) = \begin{cases} x-2 & \text{if } x \geq 2 \\ \frac{1}{x-1} & \text{if } x < 2 \end{cases}$$

$x-2$ is continuous when $x \geq 2$ and $\frac{1}{x-1}$ is continuous (except at $x=1$) when $x \leq 2$.

Let's check if $f(x)$ is continuous at $x=2$:

→ $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x-2) = 0$

→ $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{1}{x-1} = \frac{1}{2-1} = 1$

→ $f(x)$ at $x=2$ is $f(2) = 0$

Then $f(x)$ is not continuous at $x=1$ and at $x=2$.

$f(x)$ is continuous from right at $x=2$ because $\lim_{x \to 2^+} f(x) = 0 = f(2)$

$f(x)$ is neither continuous from right, nor from left at $x=1$ since $f(1)$ is not defined.

6. (15 pts) Find the derivative of the following function using the limit definition of the derivative. (without using derivative rules such as power rule, sum rule, etc.)

$$f(x) = x^2 - x - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - (x+h) - 1 - (x^2 - x - 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x} - h - \cancel{1} - \cancel{x^2} + \cancel{x} + \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 1)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h - 1 = 2x - 1$$