1. (16 pts.) Find the derivatives of the given functions.

(a) (8 pts)

$$f(x) = \frac{2\cos x}{1-x^2}$$

$$f'(x) = \frac{-2\sin x\,(1-x^2) - 2\cos x\,(-2x)}{(1-x^2)^2}$$

(b) (8 pts.)

$$f(x) = 1 + 2\sqrt{x} - 2\frac{1}{\sqrt{x}}$$

$$f(x) = 1 + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$$

$$f'(x) = 2 \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} - 2\left(-\frac{1}{2}\right) x^{-\frac{3}{2}}$$

$$= x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$$

2. (20 pts.) If a cylindrical tank holds 900 gallons of water, which can be drained from the bottom of the tank in an hour, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V(t) = (30 - t)^2 \qquad 0 \leq t \leq 30$$

Naturally the water will flow out of the tank faster when there is more water in the tank. Find the rate at which the water is flowing out of the tank (the instantaneous rate of change V with respect to t) after 10 minutes.

$$V'(t) = 2(30-t)(-1)$$

$$V'(10) = 2(30-10)(-1) = -40$$

Water is leaving the tank at 40 gal/min

3. (20 pts.) Find the minimum and maximum values of $f(x) = x^3 - 6x^2 + 9x - 8$ on the interval $[0, 2]$.

$$f'(x) = 3x^2 - 12x + 9 = 0$$

$$3(x^2 - 4x + 3) = 0$$

$$3(x-3)(x-1) = 0$$

$$\cancel{x = 3} \qquad x = 1$$

not in $[0,2]$

$$f(0) = -8 \leftarrow \text{min}$$

$$f(1) = 1 - 6 + 9 - 8 = -4 \leftarrow \text{max}$$

$$f(2) = 8 - 24 + 18 - 8 = -6$$

4. (20 pts.) Find the derivatives of the given functions.

(a) (10 pts.)

$$f(x) = \tan\sqrt{1+x^3}$$

$$f'(x) = \sec^2\sqrt{1+x^3} \cdot \frac{1}{2}(1+x^3)^{-\frac{1}{2}}\ (3x^2)$$

$$= \frac{3x^2 \sec^2\sqrt{1+x^3}}{2\sqrt{1+x^3}}$$

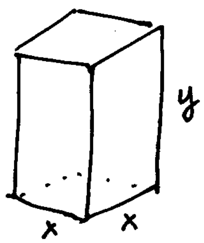
(b) (10 pts.)

$$f(x) = e^{x \ln x}$$

$$f'(x) = e^{x\ln x}\left(\ln x + x \cdot \frac{1}{x}\right)$$

$$= e^{x\ln x}(1 + \ln x)$$

5. (24 pts.) A box with a square base and without a top is to have a volume of 10 m^3. Material for the base costs \$10 per square meter. Material for the sides costs \$4 per square meter. Find the dimensions of the cheapest such container.



$$V = x^2 y = 10 \Rightarrow y = \frac{10}{x^2}$$

$$\text{Cost} = 4 \cdot xy\,(4) + x^2(10)$$
$$= 16xy + 10x^2$$

$$C(x) = 16x\left(\frac{10}{x^2}\right) + 10x^2$$

$$= \frac{160}{x} + 10x^2$$

$C(x)$ does not exist at $x=0$; but $x=0$ is not a feasable answer.

$$C'(x) = -\frac{160}{x^2} + 20x = 0$$

$$20x = \frac{160}{x^2}$$

$$x^3 = \frac{160}{20} = 8$$

$\underline{x = 2}$ then $y = \dfrac{10}{x^2} = \dfrac{10}{4} = 2.5$

Dimensions are $(2, 2, 2.5)$