

UNIVERSITY OF GEORGIA
DEPARTMENT OF MATHEMATICS
2200 CALCULUS 1
Fall 2005, Kazancı

TEST 3 11/9/05

Name : SOLUTIONS

Problem	Score	Points
1		20
2		20
3		24
4		16
5		20
Total		100

This is a 50 minute test. No books, notes or calculators are permitted. You may use the back of the page for additional space. Please show all your work on all problems. Good Luck!

1. (20 pts.) Find the derivative of the following function.

$$f(x) = x^{(e^x)}$$

$$y = x^{(e^x)}$$

$$\ln y = \ln(x^{(e^x)})$$

$$\ln y = e^x \ln x$$

$$\frac{y'}{y} = e^x \ln x + e^x \cdot \frac{1}{x}$$

$$y' = e^x \left(\ln x + \frac{1}{x} \right) y$$

$$y' = e^x x^{(e^x)} \left(\ln x + \frac{1}{x} \right)$$

2. (20 pts.)

$$e^{x-y} = xy$$

(a) (14 pts.) Find $\frac{dy}{dx}$

$$e^{x-y} (1 - y') = y + xy'$$

$$e^{x-y} - e^{x-y} y' = y + xy'$$

$$e^{x-y} - y = y' (e^{x-y} + x)$$

$$y' = \frac{dy}{dx} = \frac{e^{x-y} - y}{e^{x-y} + x}$$

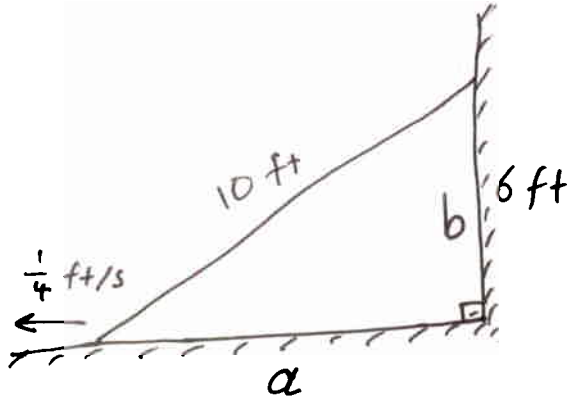
(b) (6 pts.) Find the equation of the line tangent to this curve at (1, 1).

$$y' = \frac{e^0 - 1}{e^0 + 1} = \frac{0}{2} = 0$$

$$(y - 1) = 0(x - 1) = 0$$

$$\underline{y = 1}$$

3. (24 pts.) A ladder is 10 feet long. Its top is slipping down along a vertical wall while its base is being pulled away from the base of the wall at a speed of $\frac{1}{4}$ feet/sec. How fast is the top of the ladder slipping down when it is 6 feet above the base of the wall?



$$a' = \frac{1}{4}$$

$$b' = ?$$

$$\left[\begin{array}{l} \text{When } b = 6 ; \\ a = \sqrt{10^2 - b^2} \\ = \sqrt{100 - 36} = \sqrt{64} = 8 \end{array} \right]$$

$$a^2 + b^2 = 10^2$$

$$2aa' + 2bb' = 0$$

$$2 \cdot 8 \cdot \frac{1}{4} + 2 \cdot 6 \cdot b' = 0$$

$$4 + 12b' = 0$$

$$12b' = -4$$

$$b' = \frac{-4}{12} = -\frac{1}{3} \text{ ft/sec.}$$

The top of the ladder is slipping at $\frac{1}{3}$ ft/sec.

4. (16 pts.)

$$f(x) = \frac{1}{x}$$

$$f'(x) = -\frac{1}{x^2}$$

(a) (10 pts.) Find the linear approximation of $f(x)$ at $a = 2$.

$$f(x) \approx f(a) + (x - a) f'(a)$$

$$\frac{1}{x} \approx \frac{1}{2} + (x - 2) \left(-\frac{1}{2^2}\right)$$

$$\frac{1}{x} \approx \frac{1}{2} - \frac{x - 2}{4}$$

(b) (6 pts.) Estimate the value of $\frac{1}{2.04}$ using the result of part (a).

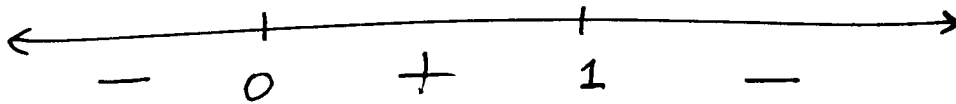
$$\frac{1}{2.04} \approx \frac{1}{2} - \frac{2.04 - 2}{4}$$

$$\frac{1}{2.04} \approx \frac{1}{2} - \frac{0.04}{4} = 0.5 - 0.01 = \underline{0.49}$$

5. (20 pts.) Find the intervals on which $f(x) = x^2 e^{-2x}$ is increasing or decreasing; indicate the local minima or maxima.

$$f'(x) = 2x e^{-2x} + x^2 e^{-2x} (-2)$$

$$= \underbrace{2x}_{\substack{\text{changes sign at } x=0 \\ \downarrow}} \underbrace{e^{-2x}}_{\substack{\text{always positive} \\ \downarrow}} \underbrace{(1-x)}_{\substack{\text{changes sign at } x=1 \\ \downarrow}}$$



$f(x)$ is increasing on $(0, 1)$ and
decreasing on $(-\infty, 0) \cup (1, \infty)$

$x=0$ is a local minima and
 $x=1$ is a local maxima.