

QUIZ 4 2/07/08

Name : Solutions

Solve the following initial value problem.

$$\frac{dy}{dt} + \sin(t)y = \sin(t) \quad y(\pi/2) = 0$$

$$\underbrace{(e^{-\cos t})}'_{\mu} = \sin t e^{-\cos t} \leftarrow \begin{array}{l} \text{found the} \\ \text{integrating} \\ \text{factor } \mu \end{array}$$

$$e^{-\cos t} \frac{dy}{dt} + \sin t e^{-\cos t} y = \sin t \cdot e^{-\cos t}$$

$$(y e^{-\cos t})' = \sin t e^{-\cos t}$$

$$\begin{aligned} y(t) e^{-\cos t} &= \int \sin t e^{-\cos t} dt \\ &= e^{-\cos t} + C \end{aligned}$$

Multiply both sides with $y(t) = \cancel{y} e^{\cos t}$:

$$y(t) = 1 + C e^{\cos t}$$

$$y(\pi/2) = 1 + C \underbrace{e^{\cos(\pi/2)}}_1 = 1 + C = 0 \Rightarrow \underline{C = -1}$$

$$\boxed{y(t) = 1 - e^{\cos t}}$$