MATH 2700 - DIFFERENTIAL EQUATIONS, Spring 2008, *Kazanci*

QUIZ 7 2/26/08

Name : SOLUTION

1. Solve the following initial value problem.

$$y'' + 4y' + 5y = 0 \qquad y(0) = 1,\ y'(0) = 0$$

$$r^2 + 4r + 5 = 0$$

$$r_{1,2} = \frac{-4 \pm \sqrt{16 - 4\cdot 5}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = -2 \pm i$$

$$y(t) = e^{-2t}\left(c_1 \cos t + i c_2 \sin t\right)$$

$$y(0) = 1\cdot(c_1 \cdot 1 + i c_2 \cdot 0) = 1 \Rightarrow \underline{c_1 = 1}$$

$$y'(t) = -2e^{-2t}(c_1 \cos t + i c_2 \sin t) + e^{-2t}(-c_1 \sin t + i c_2 \cos t)$$

$$y'(0) = -2\cdot 1\cdot(c_1 \cdot 1 + 0) + 1(-0 + i c_2 \cdot 1) = 0$$

$$\Rightarrow -2\underbrace{c_1}_{=1} + i c_2 = 0 \Rightarrow i c_2 = +2 \quad (*)$$

$$\underline{c_2 = -2i}$$

$$y(t) = e^{2t}\left(\cos t + i(-2i)\sin t\right)$$

$$\boxed{y(t) = e^{2t}(\cos t + 2\sin t)}$$

* Multiply both sides with $(-i)$.