ENGR 6101: COMPUTATIONAL ENGINEERING

Problem Set 1 *(due Thursday, 8/28)*

Show all your work on all problems. Correct answers without enough work will be graded as 0/3.

Questions:

1. (3 pts.) A computer with 4 digit precision would recognize the given values as follows:

$$\begin{aligned} \pi &\rightarrow 0.3142 \times 10^1 \\ 12347.89 &\rightarrow 0.1235 \times 10^5 \\ 0.0001231234 &\rightarrow 0.1231 \times 10^{-3} \\ 1 &\rightarrow 0.1000 \times 10^1 \end{aligned}$$

   We can emulate the algebraic operations on such a computer by doing the same operation on a higher precision machine (eg. a calculator,matlab, etc.) and than decrease the precision as shown above (rounding to the nearest decimal). But, we have to round after each basic operation, such as sum, product, square root, etc.

   (a) Consider the following expression:

$$\sqrt{x} - 3$$

   On such a computer, compute the expression above for $x = 9.01$. [1]

   (b) Note that

$$\sqrt{x} - 3 = \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}+3} = \frac{x-9}{\sqrt{x}+3}$$

   On the same computer, evaluate the last expression above, again for $x = 9.01$.

   (c) Use a higher precision machine (at least 8 digits) to compute $\sqrt{9.01} - 3$, and write down this result. Assuming that this results is "true", compute the absolute and relative errors (accurate upto at least 6 digits) for the results you obtained in part (a) and (b).

2. (3 pts.) Consider the following numerical differentiation formula:

$$f'(x) \approx \frac{\alpha f(x+h) + \beta f(x-2h)}{\gamma h}$$

   Find the appropriate values for α, β and γ for this formula to work (using Taylor series expansion). What is the order of your approximation?

3. (3 pts) Consider the following numerical differentiation formula:

$$f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

   Prove that this is a valid formula (using Taylor's expansion) and find its order.

[1] In 1(a) and 1(b), do the operations step by step. For example, in 1(a), first do the square root (on a high precision machine), and then adjust the precision to four digits. Then do the difference (again on a high precision machine), and again limit the precision of the result to four digits.

4. (3 pts) Consider the following function:

$$f(x) = x^2 + sin(x)$$

(a) Create a table by evaluating this function at $x = 0.6, 0.8, 1, 1.2$ and 1.4, accurate upto at least 8 digits[2]. Then compute the numerical approximation of the derivative at $x = 1$ using the following methods below:

i. Forward, use values $f(1)$ and $f(1.2)$.

ii. Backward, use values $f(1)$ and $f(0.8)$.

iii. Central, use values $f(0.8)$ and $f(1.2)$.

iv. New method presented in problem 3, use values $f(0.6)$, $f(0.8)$, $f(1.2)$ and $f(1.4)$.

(b) Compute the exact value by taking the derivative of this function and evaluating the derivative at $x = 1$ (accurate upto at least 8 digits). Assuming that this results is "true", find the absolute errors for all the methods above.

[2]Note that x is in radians, that is, $\sin(2\pi) = 1$.