

ENGR 6101: COMPUTATIONAL ENGINEERING

Problem Set 3 (due Friday in class, 9/18)

Questions:

1. Compute the integral

$$\int_0^{\pi/2} \sin(x) dx = 1$$

in the following ways:

- (a) (1 pts.) Using Trapezoid rule with 1 trapezoid ($x = 0, \frac{\pi}{2}$).
- (b) (1 pts.) Using Trapezoid rule with 2 trapezoids ($x = 0, \frac{\pi}{4}, \frac{\pi}{2}$).
- (c) (2 pts.) Using Richardson's extrapolation with the results in (a) and (b).
- (d) (2 pts.) Using Simpson's rule with 3 data points ($x = 0, \frac{\pi}{4}, \frac{\pi}{2}$).
- (e) (2 pts.) Write down the exact error for all of the above. Which numerical method gives the most accurate result?
- (f) (2 pts.) Prove that for the following integral

$$\int_0^{\pi/2} f(x) dx$$

(c) and (d) will always give the exact same result for any function $f(x)$.

2. Consider the following integral

$$\int_0^4 e^x dx$$

- (a) (3 pts.) Implement Simpson's method using Matlab to solve this integral. Your code should print (i) number of data points used, (ii) step size, (iii) value of the integral, and (iv) exact error¹. Print your code and include it in your solution. Name your code as `simpson.m` and submit soft copy as an e-mail attachment to `caner@uga.edu`.
- (b) (3 pts.) Find the order of Simpson's method by running your code using various step-sizes. Include the output of your runs, and show clearly the operations you did to estimate the order.

3. (3 pts.) Find the quadrature formula

$$\int_{-1}^1 f(x) dx \approx c \sum_{i=0}^2 f(x_i)$$

that is exact for all quadratic polynomials.

4. (3 pts.) Determine coefficients A_0 , A_1 , and A_2 that make the formula

$$\int_0^2 f(x) dx \approx A_0 f(0) + A_1 f(1) + A_2 f(2)$$

exact for all polynomials of degree 3.

¹Compute the error using the exact value of the integral.