ENGR 8102: COMPUTATIONAL ENGINEERING

Solution Set 1 *(due in class on Monday, 10/26)*

Questions:

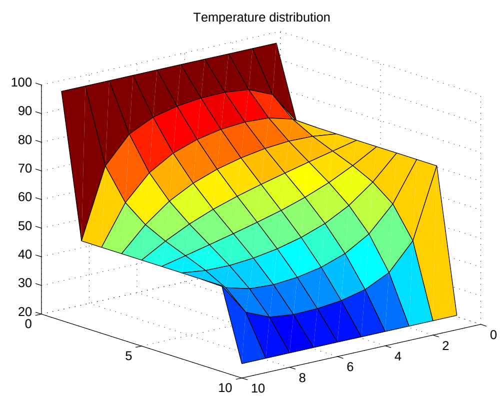
1. (7 pts.) Modify the code `laplace.m` given in class, to feature a 10x10 grid (including boundaries), instead of 5x5. Replace the zero boundary with the last two digits of your uga-ID (810-??-??XX), keep the other boundaries the same. Plot the temperature distribution and isothermal lines. Include a copy of your code, the temperature distribution matrix, and the graphs along with your solution. Submit a soft copy of your code to `caner@uga.edu`.

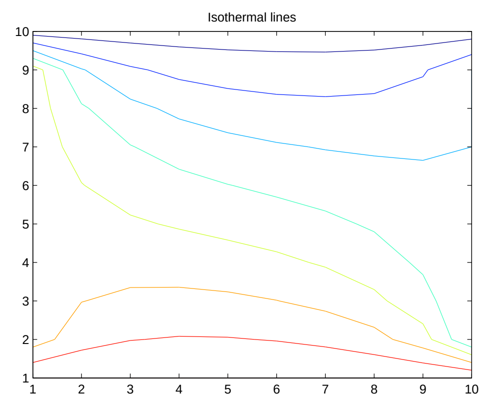
   SOLUTION. Assuming $XX = 25$:

```
function laplace

% create operator matrix m
m = diag( -4*ones(64,1) );
band1 = ones(1,63);
band1(8:8:60)=0;
m = m + diag( band1, -1 ) + diag( band1, 1 );
m = m + diag( ones(56,1), -8 ) + diag( ones(56,1), 8 );

% construct vector b using boundary conditions

bndry = [-175 -100 -100 -100 -100 -100 -100 -150 -75    0    0    0    0 ...
 0    0   -50 -75    0    0    0    0    0    0   -50 -75    0 ...
 0    0    0    0    0   -50 -75    0    0    0    0    0    0 ...
 -50 -75    0    0    0    0    0    0   -50 -75    0    0    0 ...
 0    0    0   -50 -100  -25  -25  -25  -25  -25  -25  -75]';
res = inv(m) * bndry;

% convert solution from vector to matrix format
mres = [ res(1:8)'; res(9:16)'; res(17:24)'; res(25:32)'; ...\
 res(33:40)'; res(41:48)'; res(49:56)'; res(57:64)' ];

% attach boundary

t_dist(:,1) = 75*ones(1,10);
t_dist(:,10) = 50*ones(1,10);
t_dist(1,:) = 100*ones(1,10);
t_dist(10,:) = 25*ones(1,10);
t_dist(2:9,2:9) = mres

figure;     % plot temperature distribution
surf( t_dist );
view(-220,20)
title('Temperature distribution');

figure;     % plot isothermal lines
contour( t_dist )
title('Isothermal lines')
```

   Here is the output of the code:

```
>> hw21_laplace
t_dist =
  100.0000  100.0000  100.0000  100.0000  100.0000  100.0000  100.0000  100.0000  100.0000  100.0000
   75.0000   86.1291   89.7163   90.6861   90.5187   89.5683   87.5922   83.5455   74.3145   50.0000
   75.0000   79.8000   82.0502   82.5092   81.8204   80.1623   77.2549   72.2751   63.7127   50.0000
   75.0000   76.0205   76.1754   75.4801   74.0913   72.0058   68.9900   64.5874   58.2611   50.0000
   75.0000   73.1067   71.1508   69.1444   67.0589   64.7795   62.1121   58.8233   54.7444   50.0000
   75.0000   70.2556   66.1767   62.8879   60.2205   57.9411   55.8556   53.8492   51.8933   50.0000
   75.0000   66.7389   60.4126   56.0100   52.9942   50.9087   49.5199   48.8246   48.9795   50.0000
   75.0000   61.2873   52.7249   47.7451   44.8377   43.1796   42.4908   42.9498   45.2000   50.0000
   75.0000   50.6855   41.4545   37.4078   35.4317   34.4813   34.3139   35.2837   38.8709   50.0000
   25.0000   25.0000   25.0000   25.0000   25.0000   25.0000   25.0000   25.0000   25.0000   25.0000
>>
```





2. (7 pts.) Modify the code `gauss_seidel.m` to feature a 10x10 grid (including boundaries) instead of 5x5. Replace the zero boundary with the last two digits of your uga-ID (810-??-??XX), keep the other boundaries the same. Run enough iterations until the absolute value of the difference between componentwise temperature distribution difference between (1) and (2) is bounded above by 1×10^{-2}. Plot the temperature distribution and isothermal lines. Include a copy of your code, the temperature distribution matrix, difference of temperature distributions (between Laplace and Gauss-Seidel), and the graphs along with your solution. Submit a soft copy of your code to `caner@uga.edu`. .

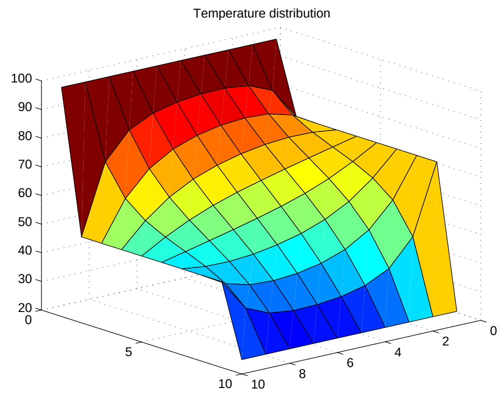
SOLUTION. Assuming $XX = 25$:

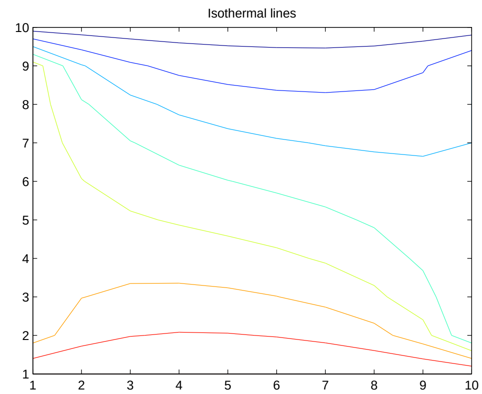
```
function gs

% create grid with boundary
T = zeros(10,10);
T(:,1)   = 75*ones(1,10);
T(:,10)  = 50*ones(1,10);
T(10,:)  = 25*ones(1,10);
T(1,:)   = 100*ones(1,10);
T_new = zeros(10,10);  % create new iteration matrix

% main iteration
error = 1;
while ( error > 0.0005 )
  for i=2:9
    for j=2:9
      T_new(i,j) = ( T(i+1,j) + T(i-1,j) + T(i,j+1) + T(i,j-1) )/4;
    end
```

```
  end
  error = max(max( abs(T(2:9,2:9) - T_new(2:9,2:9)) ));
  T(2:9,2:9) = T_new(2:9,2:9);    % update T
end
T

figure;       % plot temperature distribution
surf( T );
view(-220,20)
title('Temperature distribution');

figure;       % plot isothermal lines
contour(T)
title('Isothermal lines')
```

Here is the output of the code. Note that we had to set the approximate error a lot less to achieve the exact error of 0.01.

```
>> hw21_gs
T =
  100.0000  100.0000  100.0000  100.0000  100.0000  100.0000  100.0000  100.0000  100.0000  100.0000
   75.0000   86.1282   89.7146   90.6837   90.5161   89.5657   87.5899   83.5437   74.3136   50.0000
   75.0000   79.7982   82.0470   82.5049   81.8155   80.1574   77.2506   72.2719   63.7110   50.0000
   75.0000   76.0182   76.1711   75.4743   74.0847   71.9992   68.9842   64.5831   58.2588   50.0000
   75.0000   73.1041   71.1459   69.1378   67.0514   64.7719   62.1055   58.8183   54.7418   50.0000
   75.0000   70.2530   66.1718   62.8813   60.2130   57.9335   55.8489   53.8442   51.8906   50.0000
   75.0000   66.7366   60.4083   56.0041   52.9876   50.9021   49.5141   48.8203   48.9772   50.0000
   75.0000   61.2856   52.7217   47.7408   44.8327   43.1747   42.4865   42.9466   45.1983   50.0000
   75.0000   50.6846   41.4528   37.4055   35.4291   34.4787   34.3116   35.2820   38.8700   50.0000
   25.0000   25.0000   25.0000   25.0000   25.0000   25.0000   25.0000   25.0000   25.0000   25.0000

>> abs( T - t_dist )
ans =
         0         0         0         0         0         0         0         0         0         0
         0    0.0009    0.0017    0.0024    0.0026    0.0026    0.0023    0.0018    0.0009         0
         0    0.0018    0.0032    0.0043    0.0049    0.0049    0.0043    0.0032    0.0017         0
         0    0.0023    0.0043    0.0058    0.0066    0.0066    0.0058    0.0043    0.0023         0
         0    0.0026    0.0049    0.0066    0.0075    0.0076    0.0066    0.0050    0.0026         0
         0    0.0026    0.0049    0.0066    0.0075    0.0076    0.0067    0.0050    0.0027         0
         0    0.0023    0.0043    0.0059    0.0066    0.0066    0.0058    0.0043    0.0023         0
         0    0.0017    0.0032    0.0043    0.0050    0.0049    0.0043    0.0032    0.0017         0
         0    0.0009    0.0017    0.0023    0.0026    0.0026    0.0023    0.0017    0.0009         0
         0         0         0         0         0         0         0         0         0         0
>>
```


Temperature distribution
100
90
80
70
60
50
40
30
20
0
5
10
10
8
6
4
2
0


Isothermal lines
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10