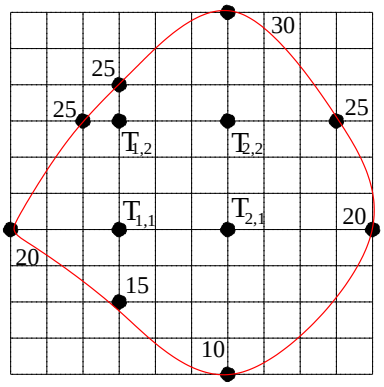
ENGR 8102: COMPUTATIONAL ENGINEERING

Problem Set 2 *(due in class on Monday, 10/26)*

Questions:

1. (7 pts.) Write down the four coupled linear equations for the following system. Then rewrite them in matrix form ($Ax = b$), where x is a vector of temperature variables, b is vector of scalars, and A is a matrix of scalars. Find the values of T_{11}, T_{12}, T_{21} and T_{22} by solving this system ($x = A^{-1}b$).



SOLUTION.

$$\begin{aligned}
\frac{1}{3}\left(\frac{20-T_{11}}{3}-\frac{T_{11}-T_{21}}{3}\right)+\frac{1}{5/2}\left(\frac{T_{12}-T_{11}}{3}-\frac{T_{11}-15}{2}\right) &= 0\\
\frac{1}{2}\left(\frac{25-T_{12}}{1}-\frac{T_{12}-T_{22}}{3}\right)+\frac{1}{2}\left(\frac{25-T_{12}}{1}-\frac{T_{12}-T_{11}}{3}\right) &= 0\\
\frac{1}{7/2}\left(\frac{T_{11}-T_{21}}{3}-\frac{T_{21}-20}{4}\right)+\frac{1}{7/2}\left(\frac{T_{22}-T_{21}}{3}-\frac{T_{21}-10}{4}\right) &= 0\\
\frac{1}{3}\left(\frac{T_{12}-T_{22}}{3}-\frac{T_{22}-25}{3}\right)+\frac{1}{3}\left(\frac{30-T_{22}}{3}-\frac{T_{22}-T_{21}}{3}\right) &= 0
\end{aligned}$$

Re-organizing these equations, we get

$$\begin{aligned}
25T_{11}-5T_{21}-6T_{12} &= 235\\
8T_{12}-T_{11}-T_{22} &= 150\\
14T_{21}-4T_{11}-4T_{22} &= 90\\
4T_{22}-T_{12}-T_{21} &= 55
\end{aligned}$$

Rewriting these equations in the form $Ax = b$ we get:

$$\begin{bmatrix} 25 & -6 & -5 & 0\\ -1 & 8 & 0 & -1\\ -4 & 0 & 14 & -4\\ 0 & -1 & -1 & 4 \end{bmatrix}\begin{bmatrix} T_{11}\\ T_{12}\\ T_{21}\\ T_{22}\end{bmatrix} = \begin{bmatrix} 235\\ 150\\ 90\\ 55\end{bmatrix}$$

Solving this equation by inverting the matrix $x = A^{-1}b$ we get

$$T_{11} = 19.0,\ T_{12} = 24.2,\ T_{21} = 18.8,\ T_{22} = 24.5$$

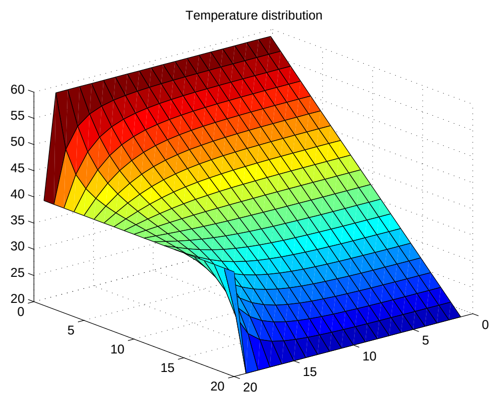
2. (7 pts.) Modify the code `gs_insulated.m` given in class, to feature an 20x20 grid (including boundaries), for the case of constant $60°C$, $40°C$ and $XY°C$ temperatures on the top, right and bottom; and insulated boundary on the left. Here, XY is the last two non-zero digits of your uga-ID[1]. Plot the temperature distribution and isothermal lines (run enough iterations so that there is no visible error). Include a copy of your code and the graphs along with your solution. Submit soft copy to `caner@uga.edu`.

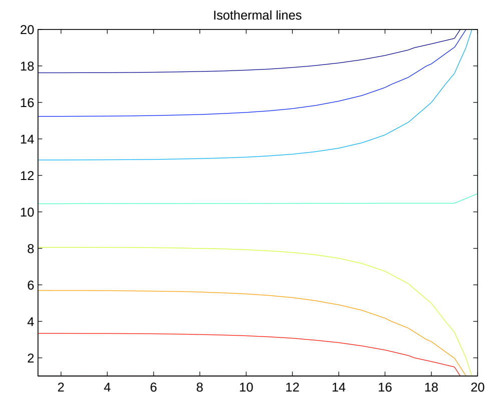
SOLUTION. Assuming that $XY = 20$:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                  %
%               60                                 %
%                                                  %
%         ----T(:,20)---                           %
%         |            |                           %
%  iso T(1,:)       T(20,:)  40                    %
%         |            |                           %
%         ----T(:,1)----                           %
%                                                  %
%               20                                 %
%                                                  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function gs_insulated

% create grid with boundary
T = zeros(20,20);
T(20,:) = 40*ones(1,20);
T(:,1) = 20*ones(1,20);
T(:,20) = 60*ones(1,20);
T_new = zeros(20,20); % create new iteration matrix

% main iteration
for n=1:600
  for i=2:19
    for j=2:19
      T_new(i,j) = ( T(i+1,j) + T(i-1,j) + T(i,j+1) + T(i,j-1) )/4;
    end
    T_new(1,i) = ( T(1,i+1) + T(1,i-1) + 2 * T(2,i) + 0 )/4;
  end
  error = abs(max(max( T(1:19,2:19) - T_new(1:19,2:19)) ));
  T(1:19,2:19) = T_new(1:19,2:19);    % update T
end

error
```

[1] Use XY if your UGA ID is 810-??-UVXY. If Y=0, then use VX. If X=Y=0, then use UV, etc.

```
figure;        % plot temperature distribution
surf( T );
view(-130,30)
title('Temperature distribution');

figure;        % plot isothermal lines
contour(T')
title('Isothermal lines')
```





3. (7 pts.) Modify the code `gs_flow.m` given in class, to feature an 20x20 grid (including boundaries), for the case of constant $XY°C$ and $20°C$ temperatures on the top and bottom; and a heat flow $Q_x = 8$ entering the system from left boundary and exiting the system from right boundary (Let h=1). Here, XY is the last two non-zero digits of your uga-ID. Plot the temperature distribution and isothermal lines (run enough iterations so that there is no visible error). Include a copy of your code and the graphs along with your solution. Submit soft copy to `caner@uga.edu`.

SOLUTION. Assuming that $XY = 60$:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                  %
%                 60                               %
%                                                  %
%           ----T(1,:)----                         %
%           |            |                         %
%  8 -> T(:,1)       T(:,20) -> 8                  %
%           |            |                         %
%           ----T(20,:)---                         %
%                                                  %
%                 20                               %
%                                                  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function gs_insulated

% create grid with boundary
```

```
T = zeros(20,20);
T(1,:) = 60 * ones(1,20);
T(20,:) = 20 * ones(1,20);

T_new = zeros(20,20);   % create new iteration matrix

% main iteration
error = 1;
while ( error > .001 )
  for i=2:19
    for j=2:19
      T_new(i,j) = ( T(i+1,j) + T(i-1,j) + T(i,j+1) + T(i,j-1) )/4;
    end
    T_new(i,1) = ( T(i+1,1) + T(i-1,1) + 2 * T(i,2) + 64 )/4;
    T_new(i,20) = ( T(i+1,20) + T(i-1,20) + 2*T(i,19) - 64 )/4;
  end
  error = max(max( abs( T(2:19,1:20) - T_new(2:19,1:20) ) ));
  T(2:19,1:20) = T_new(2:19,1:20);    % update T
end
error

figure;        % plot temperature distribution
surf( T );
view(130,15)
title('Temperature distribution');

figure;        % plot isothermal lines
contour(T)
title('Isothermal lines')
```