ENGR 6101: Computational Engineering

Solution Set 4 *(due on Friday, 10/2)*

Questions:

1. Consider the following ODE:

$$\begin{aligned} x' &= x(1-x) \\ y(0) &= 0.1 \end{aligned}$$

   (a) (3 pts.) Find the analytic solution. Note that you can do this with separation of variables.

   (b) (3 pts.) Write a Matlab code that solves this equation on the interval $[0, 8]$ using Euler method. Your code should plot both the numerical and the analytic solution on the same figure. Run your code with $h = 1$ and $h = 2$. How does the accuracy change? Name your code `ode_euler.m` and submit soft copy to `caner@uga.edu.` Include a hard copy of your code and the two graphs (for $h = 1$ and $h = 2$) along with your HW solutions.

   (c) (3 pts.) Repeat part (b) using second order Taylor-series method. In this case, name your code `ode_taylor2.m`. Again, include both hard and soft copies of your code, and a hard copy of the two graphs.

   (d) (3 pts.) Repeat part (b) using second order Runge-Kutta method. In this case, name your code `ode_rk2.m`. Again, include both hard and soft copies of your code, and a hard copy of the two graphs.

   (e) (3 pts.) Repeat part (b) using fourth order Runge-Kutta method. In this case, name your code `ode_rk4.m`. Again, include both hard and soft copies of your code, and a hard copy of the two graphs.

   (f) (3 pts.) Sort the accuracy of all four numerical methods that you've tried. Second order Taylor and second order Runge-Kutta are both second order methods. What could be the mathematical reason for the additional accuracy of the better method?

2. (3 pts.) Prove that when the fourth-order Runge-Kutta method is applied to the problem $x' = \lambda x$, the formula for advancing this solution will be

$$x(t+h) = \left[1 + h\lambda + \frac{1}{2}h^2\lambda^2 + \frac{1}{6}h^3\lambda^3 + \frac{1}{24}h^4\lambda^4\right] x(t)$$