ENGR 8102: COMPUTATIONAL ENGINEERING

Problem Set 3 *(due by noon on Wednesday, 11/9)*

Questions:

1. (4 pts.) Compute the $\mathscr{L}^1$, $\mathscr{L}^2$ and $\mathscr{L}^\infty$ norms of the following vectors and periodic functions:
   (a) $u = [-1,\ 1,\ 100]$
   (b) $v = [100,\ -100,\ 100]$
   (c) $f(\theta) = 1$ (See footnote[1])
   (d) $g(\theta) = sin(\theta)$

2. (3 pts.) Show that
$$\left\| \frac{1}{a+ib} \right\|_{\mathscr{L}^2} = \frac{1}{\|a+ib\|_{\mathscr{L}^2}}$$
   holds for any real values a and b.

3. Consider the following transport PDE:
$$u_t + 2u_x = 0$$
   (a) (7 pts.) Use Von-Neumann analysis to find the condition of convergence for the forward time, backward space scheme.
   (b) (7 pts.) Using Von-Neumann analysis, study the condition of convergence for the backward time, forward space scheme:

[1] Here are the definitions of $\mathscr{L}^1$ and $\mathscr{L}^\infty$ norms for 2π periodic functions:

$$\|f(\theta)\|_{\mathscr{L}^1} = \frac{1}{2\pi}\int_0^{2\pi} |f(\theta)|\,d\theta, \qquad \|f(\theta)\|_{\mathscr{L}^2} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [f(\theta)]^2\,d\theta}, \qquad \|f(\theta)\|_{\mathscr{L}^\infty} = \max_{0<\theta<2\pi} |f(\theta)|.$$