

UNIVERSITY OF GEORGIA
ENGR 6101 - COMPUTATIONAL ENGINEERING
Fall 2008, Kazanci

09/18/08

Name : _____

Problem	Score	Points
1		7
2		7
3		7
4		7
5		7
6		7
Total		49
		100

This is a 60 minute test. No books, notes or calculators are permitted. You may use the back of the page for additional space. Correct answers without solutions will not receive any points. Please show all your work on all problems. Good Luck!

1. (7 pts.) Write down the Taylor series for natural logarithm function about 1. Evaluate $\ln(1.1)$ by summing the first three non-zero terms in the Taylor series. Compare your result with the actual value of $\ln(1.1) = 0.095310179804$. How many accurate digits did you get?

2. (7 pts.) Prove that the following formula is a valid approximation for the second derivative (using Taylor's expansion). Find the order of this approximation.

$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

3. (7 pts.) Suppose that you are given the following table:

x	0.0	0.25	0.5	0.75	1.0
$f(x)$	1	1.2	1.6	2	3

Compute your estimate of

$$\int_0^1 f(x) dx$$

using Simpson's rule.

4. (7 pts.) Suppose that you are given a smaller table for the same function in Problem 3:

x	0.0	0.5	1
$f(x)$	1	1.6	3

Compute your best estimate of

$$\int_0^1 f(x) dx$$

using

- (a) Trapezoid rule with 1 trapezoid.
- (b) Trapezoid rule with 2 trapezoids.
- (c) Richardson's extrapolation combining the results in (a) and (b).

5. (7 pts.) Find A , B and C so that the numerical integration rule of the form

$$\int_{-1}^1 xf(x) dx \approx Af(-1) + Bf(0) + Cf(1)$$

is exact for all $f(x)$ polynomials of maximum degree m . What is m ?

6. (7 pts.) Find $x(0.1)$ by solving the differential equation

$$\begin{cases} x' = -tx^2 \\ x(0) = 2 \end{cases}$$

with one step of the Taylor-series method of order 2.