

# Mathematical Biology, Kazanci

## Homework Assignment 3 (due 3/24/08)

1. Consider the simple epidemic with an exposed (latent) state  $E$  between susceptible and infective, whose members have contracted the disease but are not yet infectious.

- (a) Explain the equations

$$\frac{dS}{dt} = -aIS, \quad \frac{dE}{dt} = aIS - bE, \quad \frac{dI}{dt} = bE$$

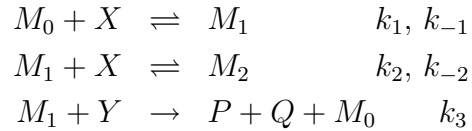
for the  $SEI$  disease.

- (b) Reduce the system into two variables,  $S$  and  $I$  using conservation rules.
  - (c) Find all fixed points and analyze their stability using linearization (Do not use a computer). Classify them as stable node, unstable node, stable spiral, unstable spiral, saddle or center.
  - (d) Depending on the parameters ( $a$ ,  $b$  and  $m$  where  $m = S + I + E$ ), how many cases are possible? Sketch the phase-plane (nullclines, direction fields) of all possible configurations (You can use a computer if you prefer, but it is not required).
2. Consider the following predator prey system:

$$u' = ru(1-u) - \frac{puv}{k+u}$$
$$v' = \frac{puv}{k+u} - dv$$

- (a) Explain why the predation term ( $\frac{puv}{k+u}$ ) makes sense.
  - (b) Explain the physical meanings of each parameter:  $r$ ,  $d$ ,  $p$ ,  $k$ .
  - (c) Assume  $r = 2$ ,  $p = 0.5$ ,  $d = 0.2$ ,  $k = 0.5$ . Write an XPP code for this system. Plot nullclines, and the scaled direction field (on the same figure). Print this figure and your code, and include them in your HW solutions.
  - (d) We are interested in the effects of an increase in the the predation rate ( $p$ ) on prey population ( $u$ ). To accomplish this, use XPPAUT to plot the bifurcation diagram for  $0.5 < p < 3$ . I suggest using the following numerical parameters:  $NPr=500$ ,  $Nmax=2000$ ,  $Par\ Max=3$ ,  $Dsmax=0.05$ . Print and include this diagram in your HW solutions.
3. What is the Lorenz Attractor? Use ode45 in Matlab to create the Lorenz attractor. Print this figure and your codes, and include them in your HW solutions.

4. Consider the following system:



This system is called a *substrate inhibited reaction*.

- Identify the substrate, enzyme, products, active (capable to produce product) and passive (not capable to produce product) complex.
- Write down the complete ODE system.
- Rewrite the ODE using conservation. Make quasi-steady state assumptions to show that

$$m_0 = m_1 \frac{k_{-1} + k_3 y}{k_1 x}, \quad m_2 = k_2 x \frac{m_1}{k_{-2}}$$

- Use these results to show that

$$m_1 = \frac{(E)k_1 x}{k_{-1} + k_3 y + k_1 x \left(1 + \frac{k_2}{k_{-2}} x\right)}$$

where  $E = M_0 + M_1 + M_2$ .