
Systems of Discrete-Time Equations

Lecture 3

Caner Kazanci

Mathematical Biology
University of Georgia
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Alternatives to Logistic Equation

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Ricker model:

$$x_{n+1} = e\left[r\left(1 - \frac{x_n}{K}\right)\right]x_n$$

Multistep Discrete-Time Equations

Fibonacci series:

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Explicit form:

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right]$$

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