Ecological Network Decomposition

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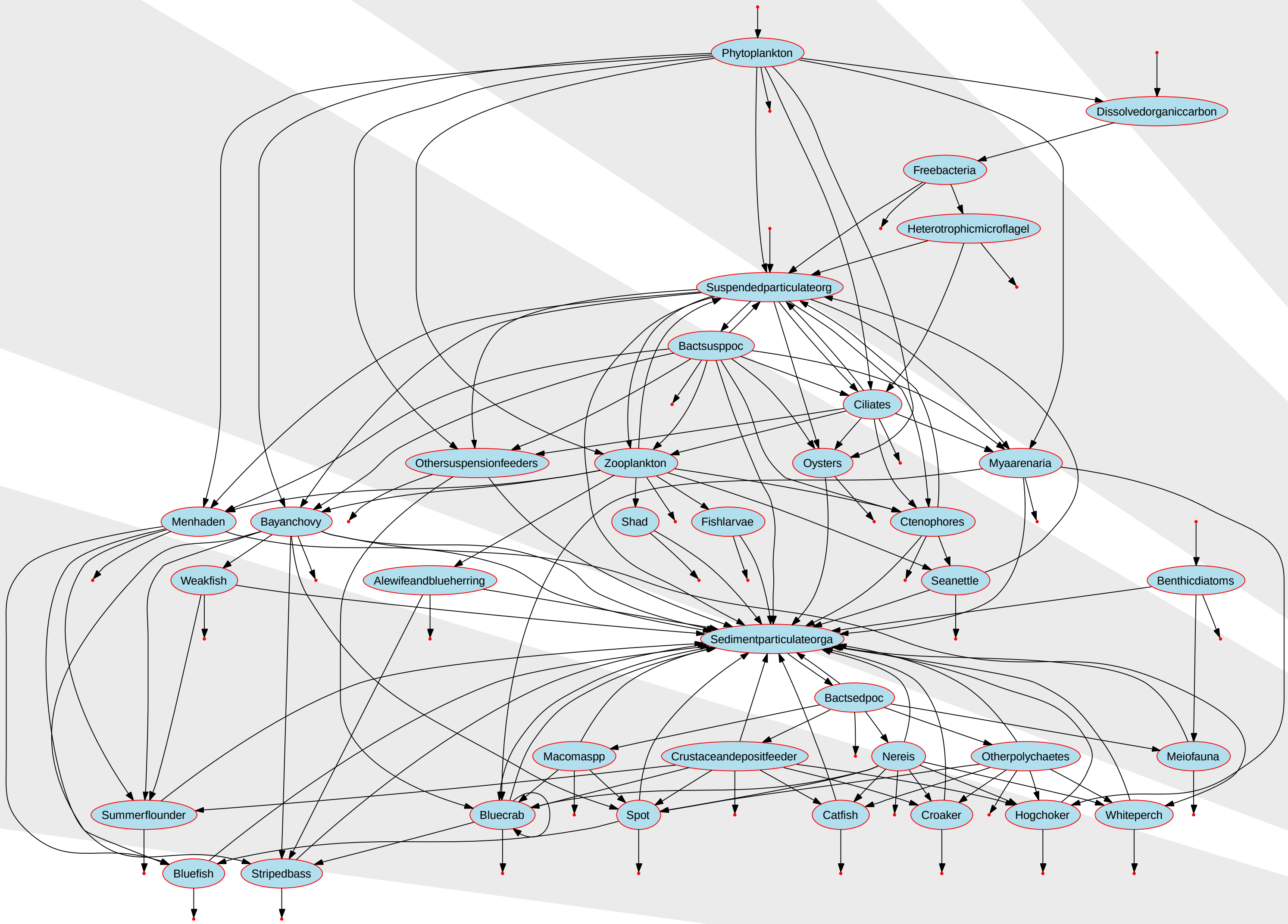
Ecosystems Models

- Ecosystems are modeled as digraphs (flows among compartments)
- Nodes, Stocks, Compartments:
  - accumulated organic matter, hundreds of species,...
- Edges, Links, Flows:
  - energy, nutrients, biomass, C, N, P, O...
- Flow currencies are conserved
- Non spatial

Chesapeake Bay ecosystem



Understanding ecosystems

Important ecological function indicators?
Ecological goal functions?
Ecosystem health?

- ◊ Sum of throughflows (Odum)
- ◊ Cycling index (Finn)
- ◊ Dominance of indirect effects (Patten)
- ◊ Ascendancy (Ulanowicz)
- ◊ Exergy (Jørgensen)
- ◊ New definitions, formulations?...

Nomenclature

x_i : Storage value at compartment i.

F_{ij} : Flow from compartment j to compartment i per unit of time.

z_i : Input from the environment to i.

y_i : Output to the environment from compartment i.

T_i : Throughflow at compartment i, $T = \sum_{i=1}^{n} f_{ki} + z_k = \sum_{i=1}^{n} f_{ik} + y_k$

G : Normalized flow matrix, $G_{ij} = F_{ij}/T_j$

τ_i : Turnover rate at compartment i, T_i/x_i

C : Partial turnover rate matrix, $C_{ij} = F_{ij}/x_j$

TST : Total System Throughflow, $TST = \sum_{i=1}^{n} T_i$

Network Environ Analysis

- Throughflow Analysis (N)
  N_{ij} represents the amount of throughflow generated at i for a unit input into j.

$$N = (I - G)^{-1} \quad Nz = T$$

- Storage Analysis (S)
  S_{ij} represents the amount of storage generated at i for a unit input into j.

$$S = -C^{-1} \quad Cz = x$$

- Utility Analysis (U)

$$U = (I - D)^{-1} \quad D_{ij} = \frac{F_{ij} - F_{ji}}{T_i}$$

Some network measures

- Finn's cycling index

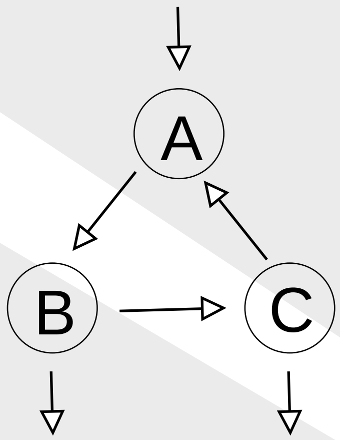
$$FCI = \sum_{i=1}^{n} \frac{T_i}{TST} \frac{N_{ii} - 1}{N_{ii}}$$

- Indirect effects ratio

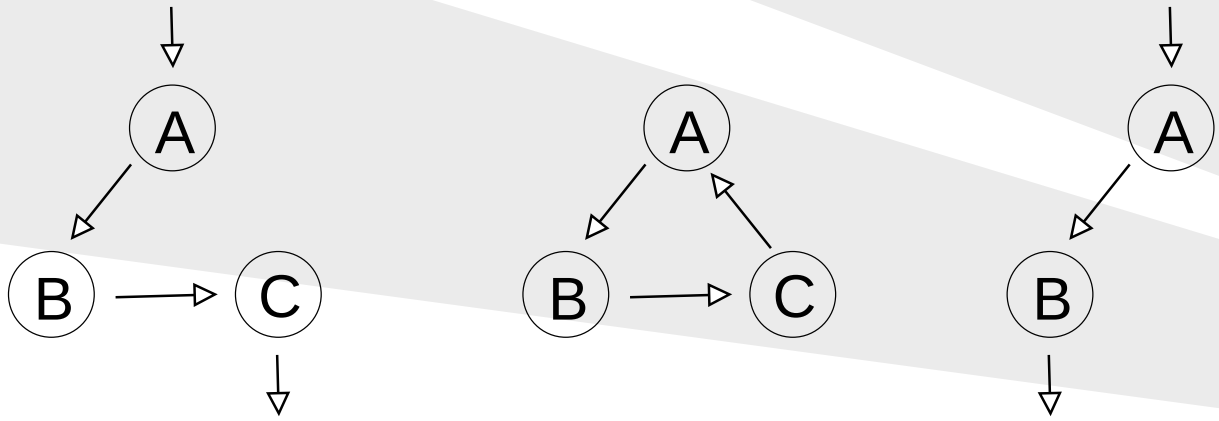
$$\frac{i}{d} = \frac{\sum_{i,j} \sum_{n=2}^{\infty} G_{ij}^{n}}{\sum_{i,j} G_{ij}} > 1$$

$$I + G + G^2 + G^3 + G^4 + \cdots = (I - G)^{-1} = N$$

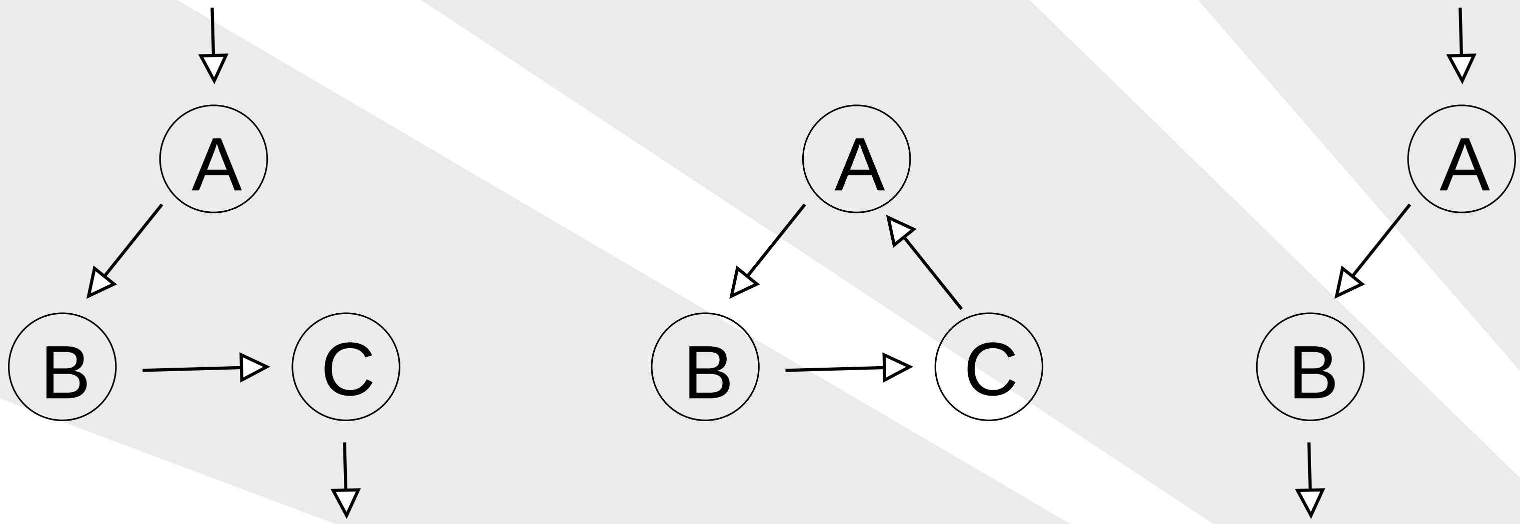
Decomposing ecosystems



Fluxes are defined as the smallest subnetworks that can sustain themselves:

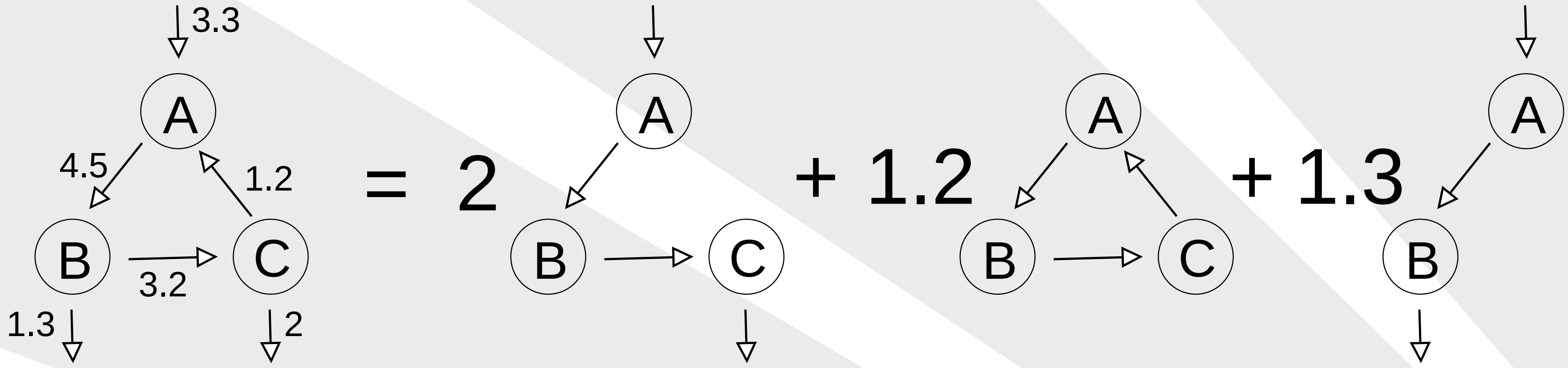


Fluxes



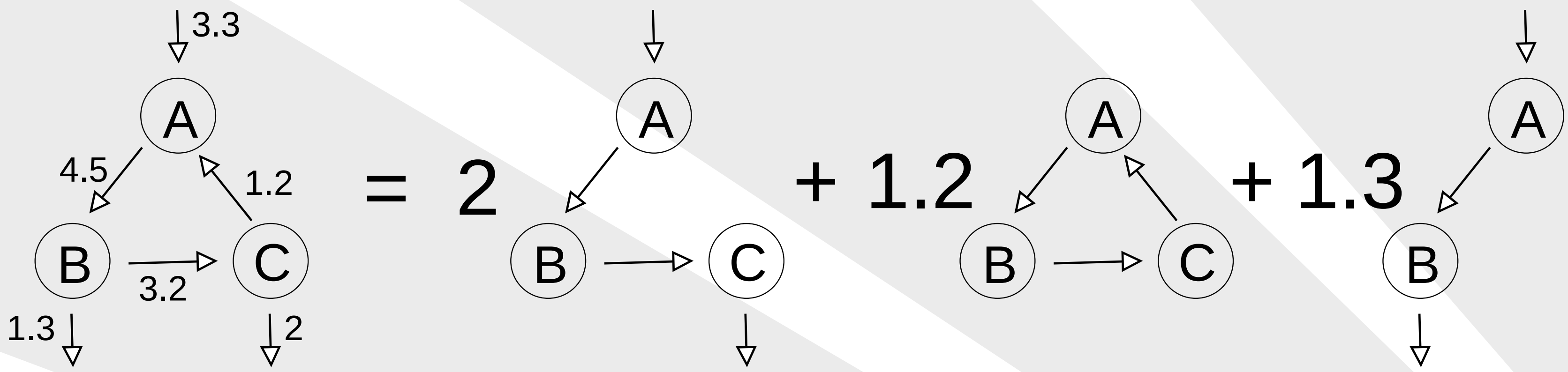
- Fluxes are either simple chains, or simple cycles.
- They either contain one input and one output, or no inputs or outputs at all.
- Fluxes can be represented by a vector of zero's and one one's. The size of this vector is equal to the number of flows in the original system.

Functional decomposition



For any ecosystem model at steady-state, is there a unique set of flux coefficients that correspond to the original network?

Functional decomposition



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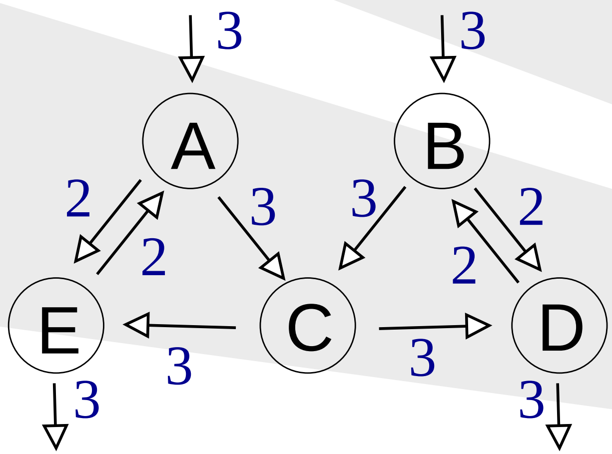
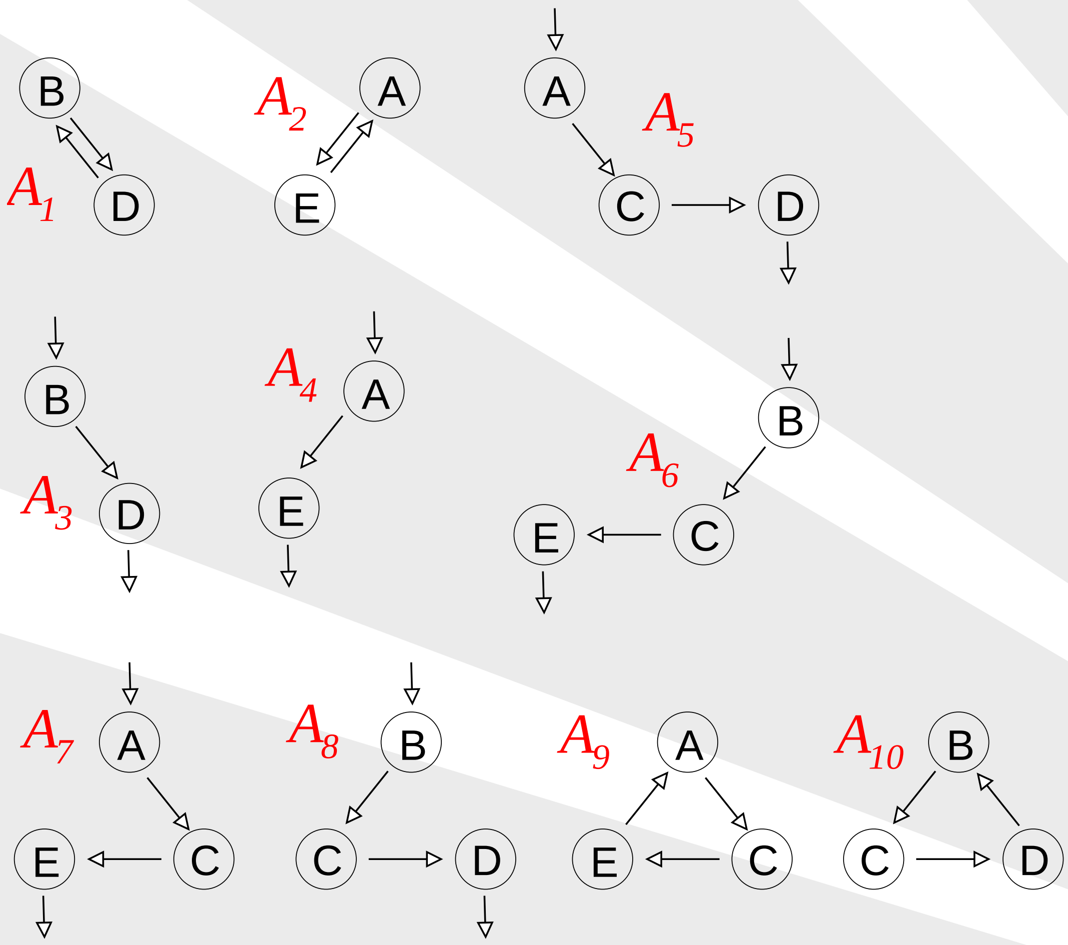
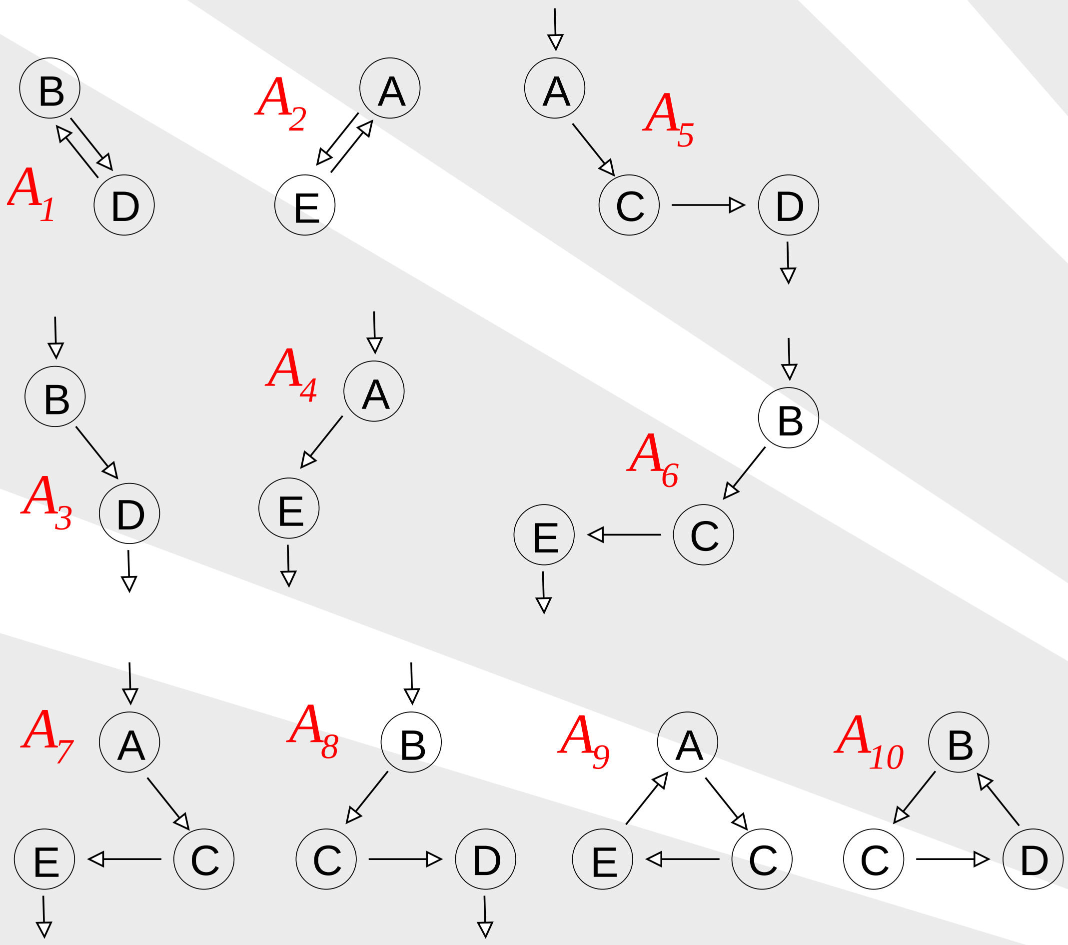


Fluxes

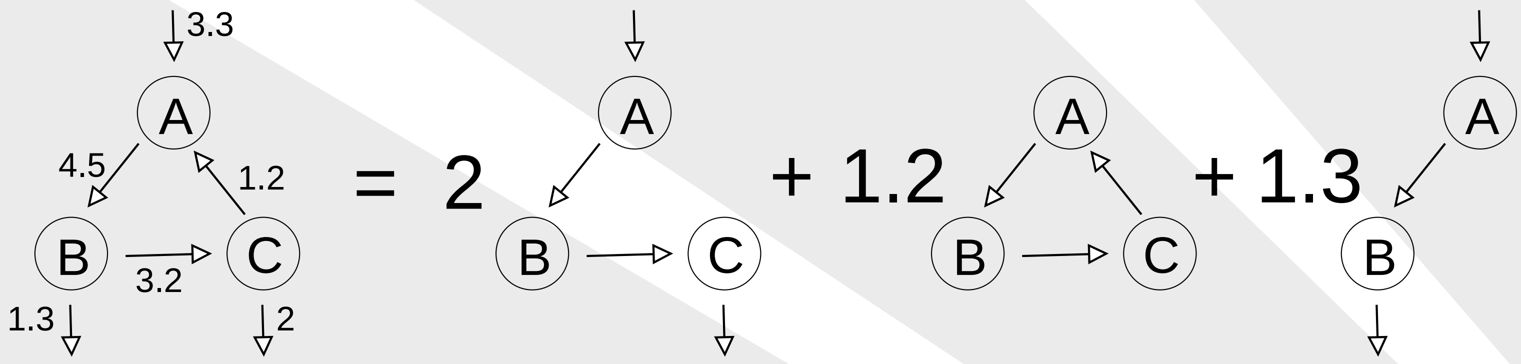


Fluxes



$$F = 2A_1 + 2A_2 + 3A_7 + 3A_8 = 2A_1 + 2A_2 + 3A_5 + 3A_6$$
$$= A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + A_9 + A_{10}$$

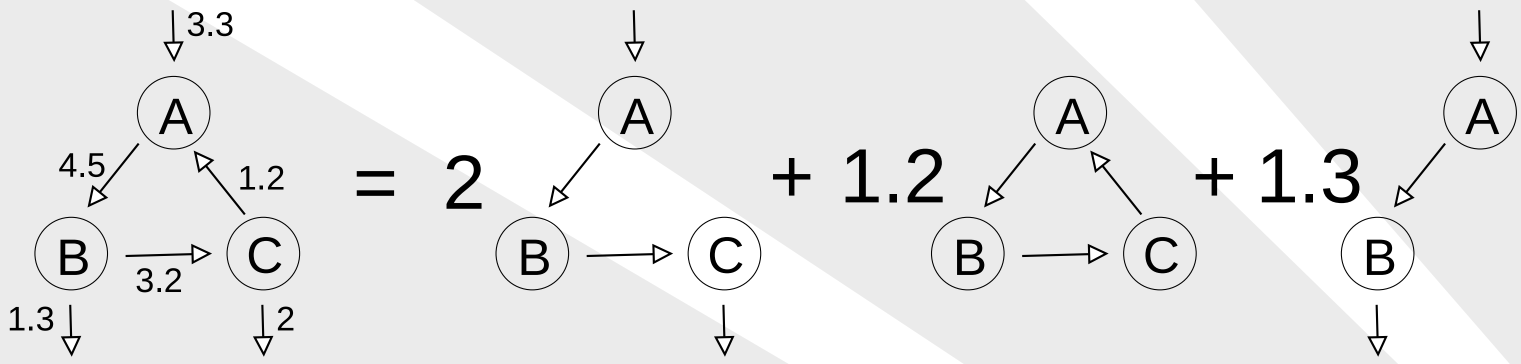
Questions



$$F = \alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_n X_n$$

- How can we compute α?

Questions

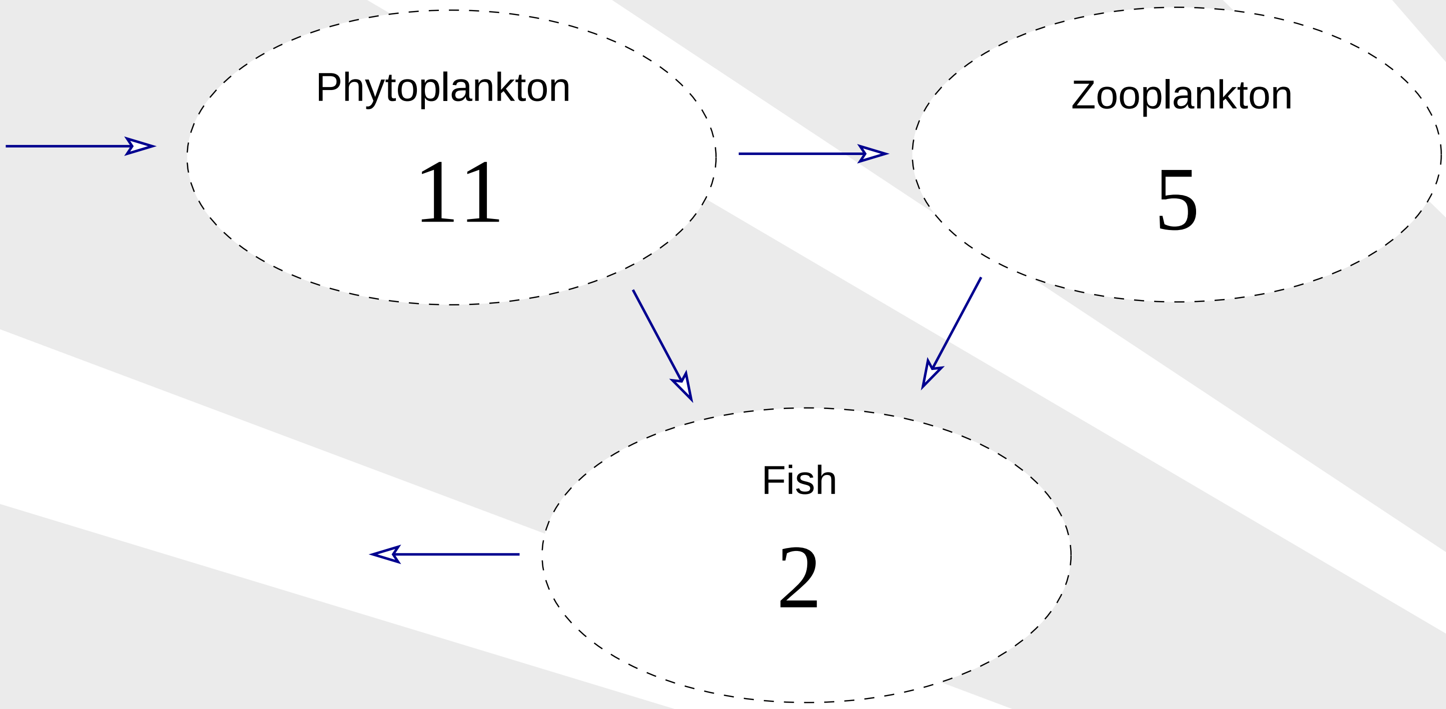


$$F = \alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_n X_n$$

- How can we compute α?
- Let $\mathcal{P}(F)$ be a systemwide property of ecological network F. Can we find operators $\odot$, $\oplus$ such that

$$F = \alpha_1 \odot X_1 \oplus \alpha_2 \odot X_2 \oplus \cdots \oplus \alpha_n \odot X_n$$

Network Particle Tracking (NPT)



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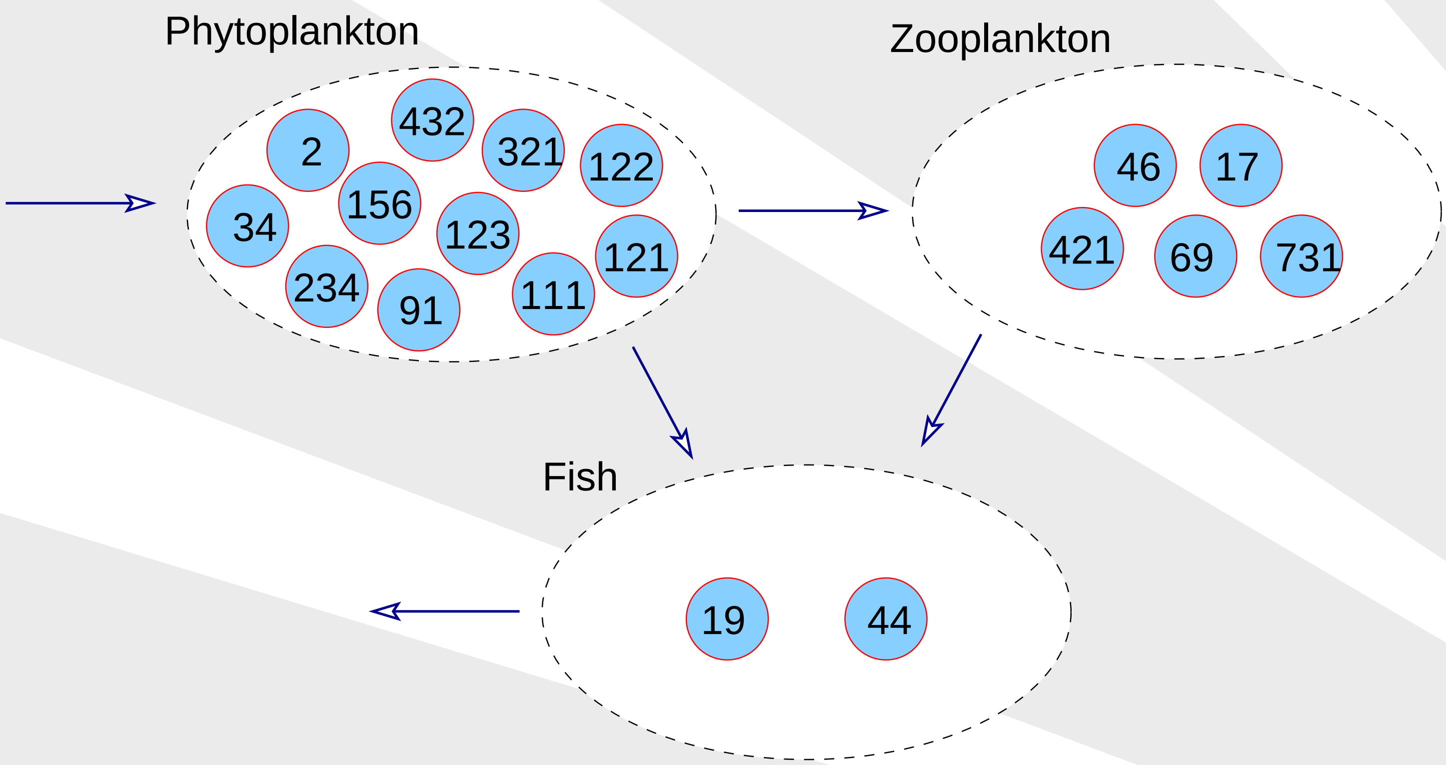

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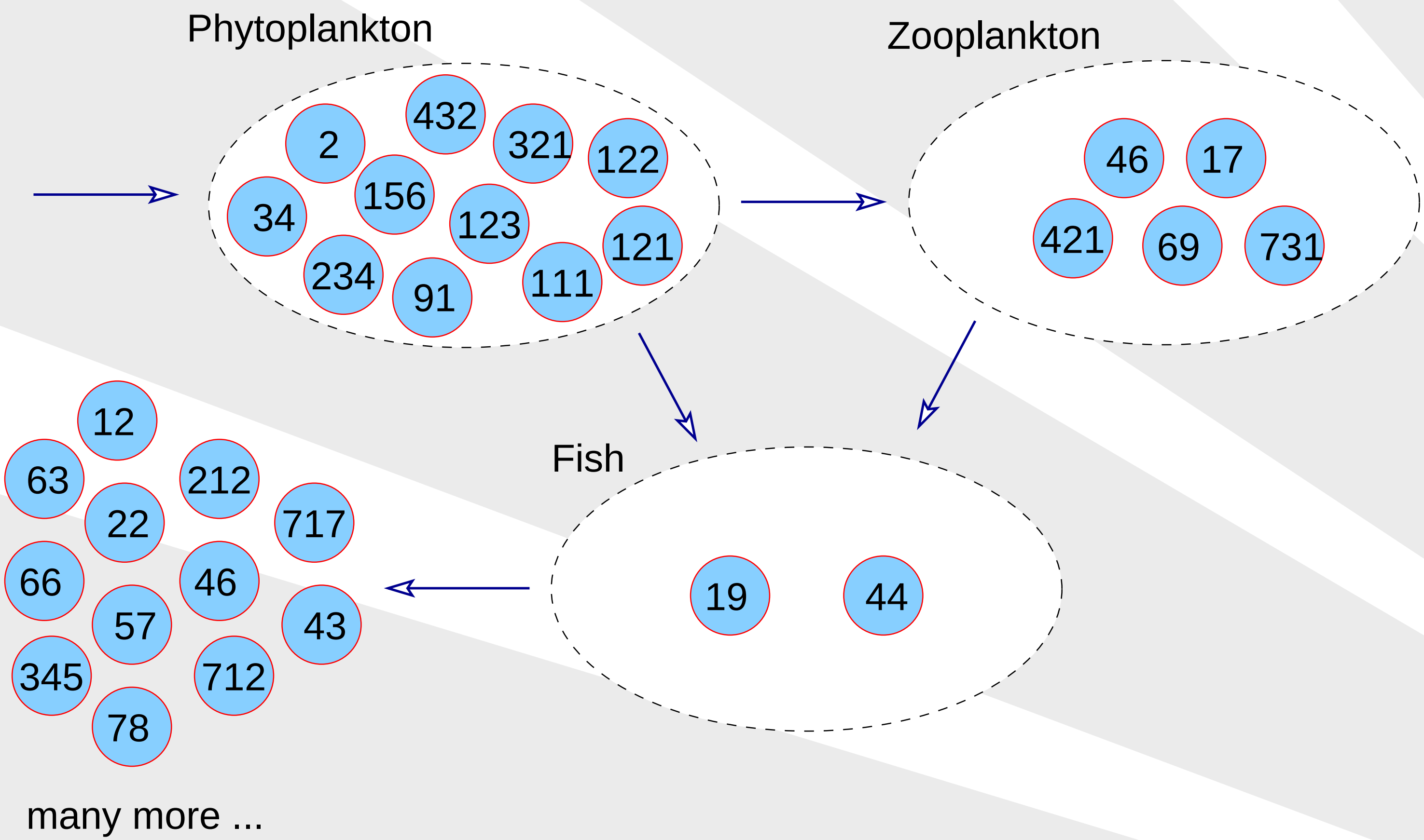

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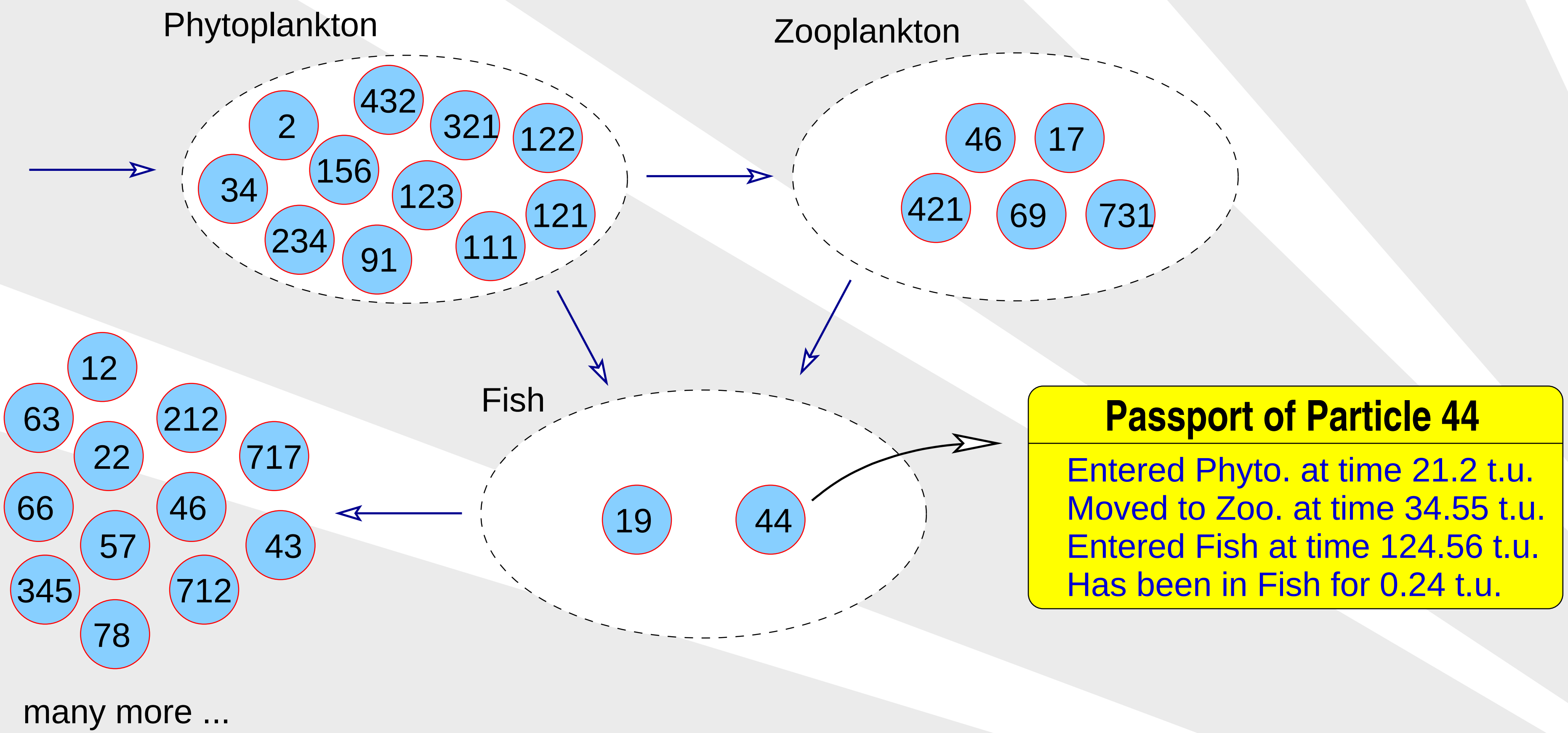

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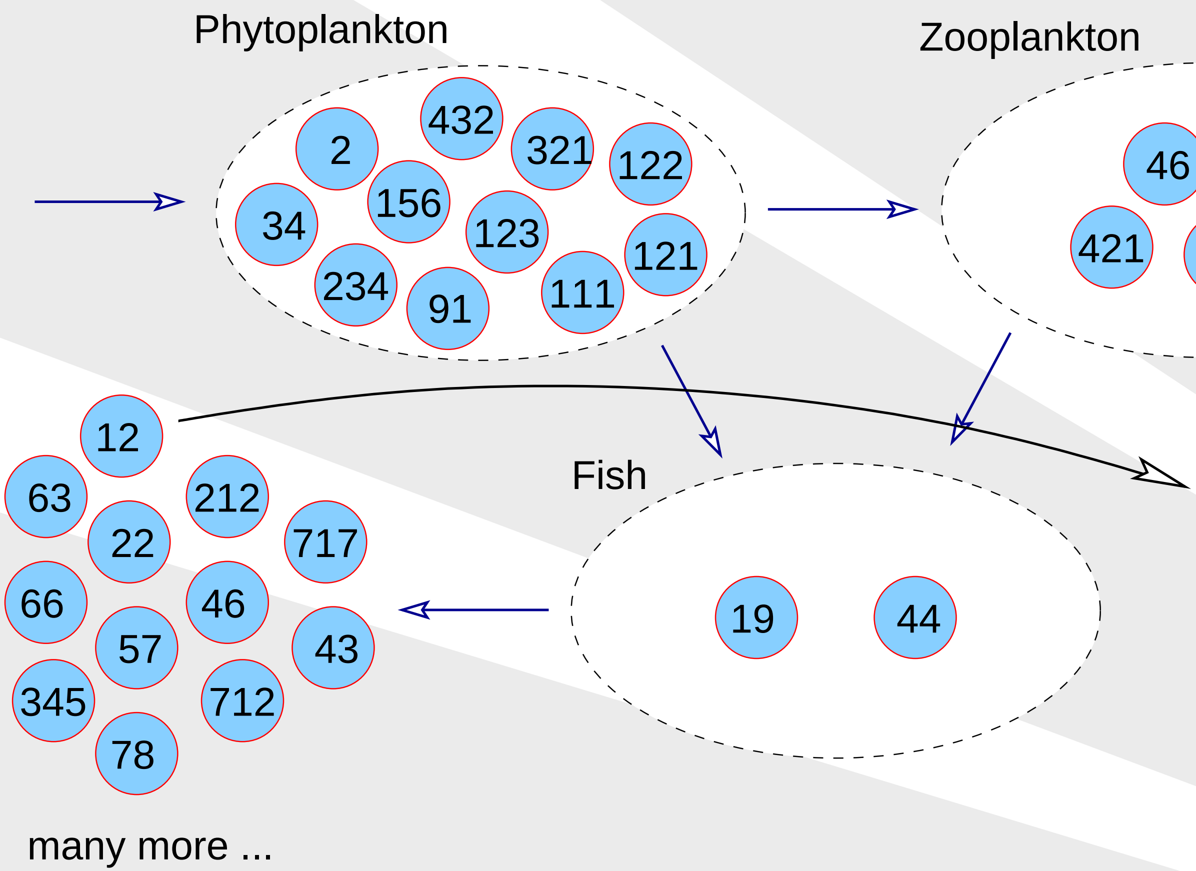
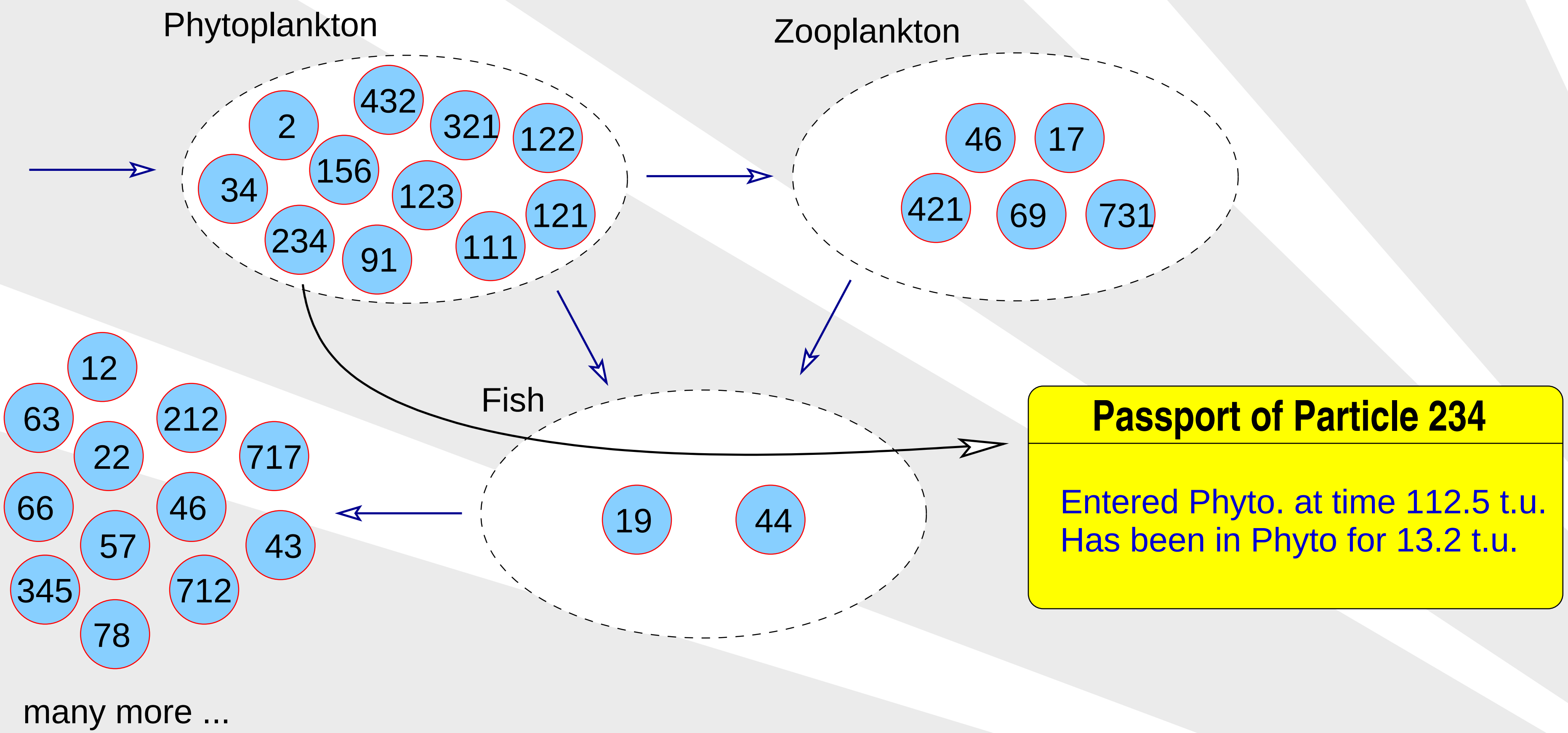


Passport of Particle 12

Was initially at Phyto.
Moved to Fish at time 67.6 t.u.
Exited the system at time 91.1 t.u.

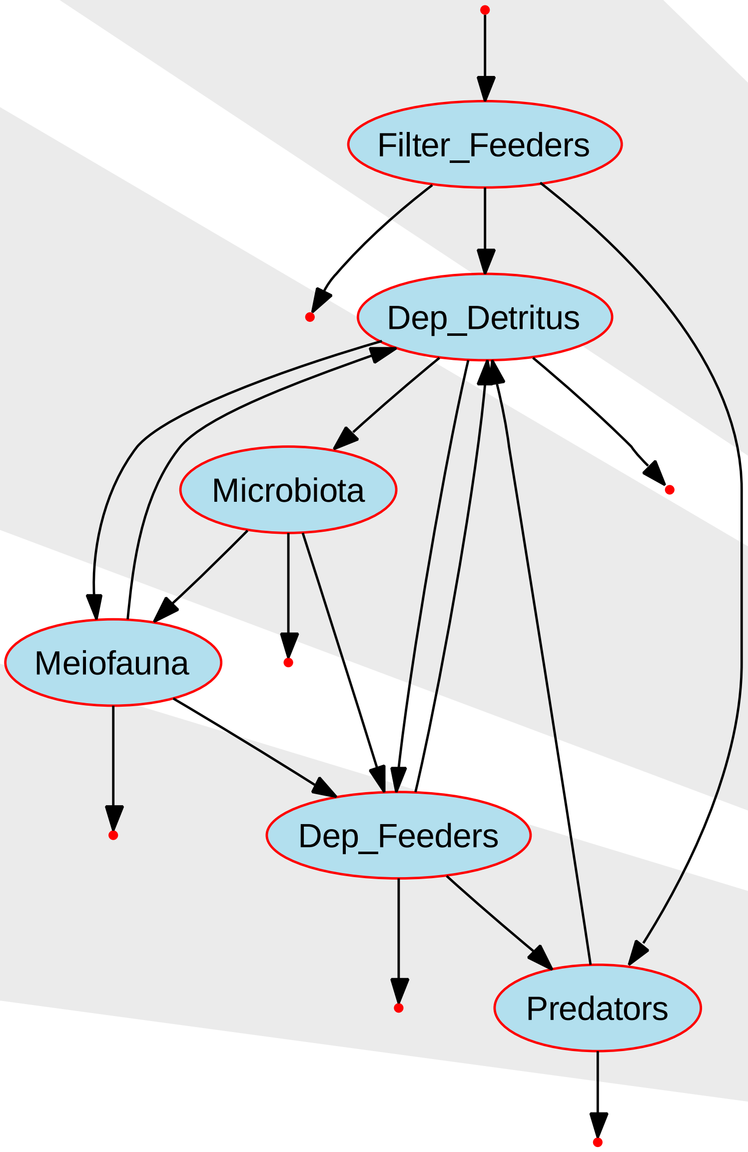
Network Particle Tracking (NPT)



What is NPT?

- An individual based algorithm
- Stochastic
- Applicable to network models with conservative flow material
- Derives all rules from the ODE representation
- Results are compatible with the ODE representation

NPT output


Filter_Feeders
Dep_Detritus
Microbiota
Meiofauna
Dep_Feeders
Predators

NPT output

Particle 144
Pathway: * → 1 → 2 → 4 → 5 → 6 → *
Flow time: 0.5 9.7 16.8 20.4 27.2 34.8
Residence time: 9.2 7.1 3.6 6.8 7.6

Particle 145
Pathway: * → 1 → 2 → 5 → 2 → 4 → *
Flow time: 1.2 10.3 19.8 28.0 33.5 41.6
Residence time: 9.1 9.5 8.2 5.5 8.1

Particle 146
Pathway: * → 1 → 2 → 4 → 2 → 3 → 4 → 5 → 6 → *
Flow time: 2.3 9.2 13.4 17.2 24.3 33.5 39.6 44.1 50.8
Residence time: 6.9 4.2 3.8 7.1 9.2 6.1 4.5 6.7

Particle 147
Pathway: * → 1 → 2 → 3 → 4 → 2 → 3 → *
Flow time: 2.9 11.8 19.7 21.8 26.6 31.7 32.9
Residence time: 8.9 7.9 2.1 4.8 5.1 1.2

Decomposition