A short summary of throughflow analysis

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x_i	:	Storage value at compartment i.
F_{ij}	:	Flow from compartment j to compartment i per unit of time.
z_i	:	Input from the environment to i.
y_i	:	Output to the environment from compartment i.
T_i^{in}	:	Throughflow (sum of inflows) into ompartment i
T_i^{out}	:	Throughflow (sum of outflows) out of ompartment i

In general, the following differential equation holds for each compartment

$$\dot{x}_k = T_k^{in} - T_k^{out}$$

where T^{in} and T^{out} are functions of time, and

$$\begin{aligned} T_k^{in} &= \sum_{i=1}^{n} f_{ki} + z_k \qquad (1) \\ T_k^{out} &= \sum_{i=1}^{n} f_{ik} + y_k \end{aligned}$$

At steady state, we have

$$T^{in} = T^{out} = T$$

Replacing T^{in} by its definition (1), we get

$$\sum_{i=1}^{n} f_{ki} + z_k = T_k \qquad (2)$$

We define G, the flow matrix normalized with respect to throughlows (T), as follows:

$$g_{ik} = \frac{f_{ik}}{T_k} \qquad (3)$$

Combining equation (2) with definition (3), we get

$$z_k = T_k - \sum_{i=1}^{n} g_{ki} T_i$$

Using matrix notation, the equation above can be expressed as follows:

$$z = (I - G)\,T$$

where G has zeros on its diagonals. Assuming $I - G$ is invertible, we define the *throughflow matrix* N as follows:

$$N z = T, \quad N = (I - G)^{-1}$$

Note that

$$N = I + G + G^2 + G^3 + \cdots = (I - G)^{-1}$$