Math 3200 Exam 1
Thursday, September 18, 2003

*Problems 2 and 4 are proofs, which require careful writing in complete sentences. In problems 1, 3, and 5 very brief solutions are OK.*

1. (15 points) Solve the following counting problem and give reasons for your answers: A class of 25 students has 16 freshmen and 12 physics majors.

(a) What is the minimum possible number of freshman physics majors in the class?

(b) What is the maximum possible number of freshman physics majors in the class?

(c) If everyone in the class is a freshman or a physics major, how many freshman physics majors are there in the class?

2. (25 points) Prove: For all sets $A$, $B$, and $C$,

$$A - (B \cup C) \subseteq (A - B) \cap (A - C).$$

Use only the definitions of the symbols $\cup$, $\cap$, $-$, and $\subseteq$. (Do not use the laws of Boolean algebra.)

3. (15 points) Determine whether the following equation is true for all sets $A$, $B$, and $C$, by using Boolean algebra to simplify the equation. Show all of your work. (You do not have to give a proof or a counterexample.)

$$C \cup ((A \cap B)' \cap (B \cup C)) = (A \cap C) \cup (A' \cap (C' \cap B')')$$

4. (25 points) Prove: For all nonnegative integers $n$,

$$1 + \frac{1}{2} + \cdots + \frac{1}{2^n} = 2 - \frac{1}{2^n}.$$

Give a reason for each step of your proof.

5. (20 points)

(a) State the binomial theorem. (Don't prove it.)

(b) State Pascal's theorem for the binomial coefficients. (Don't prove it.)

(c) Find the first four coefficients $A$, $B$, $C$, $D$ of the expansion of $(x + y)^{50}$:

$$(x + y)^{50} = Ax^{50} + Bx^{49}y + Cx^{48}y^2 + Dx^{47}y^3 + \cdots .$$