

Math 3200 Final Exam
Tuesday, December 16, 2003
8:00 - 11:00 am

1. Prove: For all sets A , B , and C ,

$$B - (A \cap C) = (B - A) \cup (B - C).$$

Use only the definitions of the symbols \cup , \cap , $-$, and $=$. (Do not use the laws of Boolean algebra.)

2. Prove: For all positive integers n ,

$$1 + 2 + \cdots + n = \frac{1}{2}n(n + 1).$$

Give a reason for each step of your proof.

3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 4x^2 - x^4$. Find $f_*([-1, 2])$ and $f^*([0, 3])$.

4. Prove: For all integers a , b , c , d , and all natural numbers n , if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$.

5. Solve each of these equations for x . Show your work. If the equation has no solutions, prove it. If the equation has infinitely many solutions, describe all of the solutions.

(a) $15x \equiv 7 \pmod{42}$

(b) $91x \equiv 26 \pmod{1079}$

6. (a) What does it mean for two sets to have the same cardinality?

(b) Prove that the set of integers and the set of natural numbers have the same cardinality.