Math 3200 Homework 2, due Thursday, August 28, 2003

We have discussed the following counting formulas for sets:

$$\#(A \cup B) = \#(A) + \#(B) - \#(A \cap B).$$

$$\#(A \cup B \cup C) = \#(A) + \#(B) + \#(C) - \#(A \cap B) - \#(A \cap C) - \#(B \cap C) + \#(A \cap B \cap C)$$

In problems 2, 3, and 4, *explain* how you use these counting formulas (or the analogous area formulas) for sets to get your answers.

1. A class of 36 students has 20 sophomores and 26 math majors.
   (a) What is the minimum possible number of sophomore math majors in the class?
   (b) What is the maximum possible number of sophomore math majors in the class?
   (c) If everyone in the class is a sophomore or a math major, how many sophomores math majors are there in the class?

2. The requirement to get a freshman scholarship at Tech is to have an A average in high school or to score at least 1400 on the SAT. Of the freshmen who got scholarships this year, 356 have an A average, 250 scored at least 1400 on the SAT, and 125 freshmen had both an A average and a score of at least 1400 on the SAT. How many freshmen got scholarships?

3. A triangle X of area 3 and a triangle Y of area 4 are contained in a triangle Z of area 6. What is the minimum possible area of the overlap of the triangles X and Y? Construct an explicit example to show that this overlap can actually have the minimum possible area.

4. Polygon X has area 10, polygon Y has area 12, and polygon Z has area 15. Suppose the area common to polygons X and Y is 2, the area common to polygons Y and Z is 2, and the area common to polygons X and Z is 2. What are the maximum possible total area and the minimum possible total area covered by the three polygons? Construct examples where the maximum and minimum areas actually occur.

5. Draw Venn diagrams for the following sets.

a. $A \cap (B \cup C)$

b. $A \cup (B \cap C)$

c. $(A \cap B) \cup (A \cap C)$

d. $A - (B \cup C)$

e. $(A - B) \cup C$

6. Using Venn diagrams, determine whether each of the following statements is true for all sets A, B, C. For each statement give a *proof* or a *counterexample.*

a. $A \cup B \subseteq A \cap B$.

b. $(A \cap B) - C = A \cap (B - C)$.

c. $A \cap (B \cup C) = (A \cap B) \cap (A \cap C)$.

d. If $A \cup B = A$, then $A = B$.

e. If $B - A = B$, then $A \cap B = \emptyset$.