Classification of surfaces with boundary

The classification of compact connected surfaces without boundary can be used to classify compact connected surfaces with boundary.

We have discussed the proof that two compact connected surfaces without boundary are homeomorphic if and only if they have the same orientation type and the same Euler characteristic. For orientable compact connected surfaces without boundary the Euler characteristics which occur are the integers of the form $2-2n$, $n=0,1,2,\ldots$. For nonorientable compact connected surfaces with boundary the Euler characteristics which occur are the integers of the form $2-n$, $n=1,2,\ldots$.

Two compact connected surfaces possibly with boundary are homeomorphic if and only if they have the same orientation type, the same Euler characteristic, and the same number of boundary components. If a surface has δ boundary components, $\delta=0,1,2,\ldots$, then a surface without boundary is obtained by glueing disks onto each of the boundary components. In other words, every compact connected surface with boundary is obtained by removing a finite number of open disks from a compact connected surface without boundary. Removal of an open disk from a surface decreases its Euler characteristic by 1.

Example: What are all the compact connected surfaces (with or without boundary) which have Euler characteristic 0? They are: a sphere with two open disks removed (i.e. an annulus), a projective plane with one open disk removed (i.e. a Möbius strip), a torus, or a Klein bottle.