MATH 4000/6000 Exam 2
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Each problem is worth 20 points.

1. Find the fourth roots of $a = -2\sqrt{3} - 6i$. (You may write the answer using trig functions.)

2. Let F be a field. Prove that if $f(x) \in F[x]$ is a polynomial of degree 2 or 3, then $f(x)$ is irreducible in $F[x]$ if and only if $f(x)$ has no roots in F.

3. Decide whether each of the following polynomials $f(x) \in F[x]$ is irreducible. If it is irreducible explain why. If it is not irreducible, give a factorization into irreducible factors.

   (a) $x^3 + \overline{2}x + \overline{5} \in \mathbb{Z}_7[x]$

   (b) $x^4 + x^3 + x + \overline{2} \in \mathbb{Z}_3[x]$

4. True or false? Give a proof or a counterexample:

   (a) $\mathbb{Q}[\sqrt{10}] \subset \mathbb{Q}[\sqrt{2}, \sqrt{5}]$

   (b) $\mathbb{Q}[\sqrt{2}, \sqrt{5}] \subset \mathbb{Q}[\sqrt{10}]$

5. Find the splitting field F of $x^4 + x^2 + 1 \in \mathbb{Q}[x]$. Write $F = \mathbb{Q}[\alpha]$ for some $\alpha \in \mathbb{C}$.