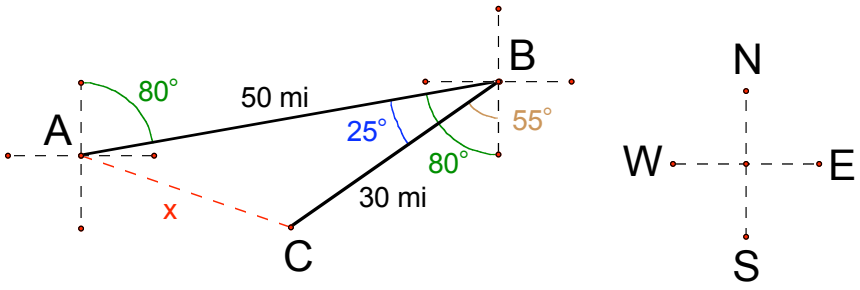
MATH 5200/7200 Exam 3 Solutions
12/5/08

1. A cruise ship leaves an island and sails 50 miles at a bearing of North 80 degrees East. It then sails 30 miles at a bearing of South 55 degrees West and stops for the night. How far is the ship from the island?

Let A be the starting point of the ship, let B be the turning point, and let C be the stopping point. Since the compass bearing of the ray AB is North 80° East, the compass bearing of the opposite ray BA is South 80° West (alternate interior angles of a transversal). Since the ray BC has bearing South 55° West, the angle ABC is 80° – 55° = 25°.



We use the law of cosines to find the distance x = AC:

$x^2 = 50^2 + 30^2 - 2(50)(30)\cos(25°)$
$= 2500 + 900 - 3000(0.906307787) = 681.0766389$
$x = 26.09744507$

The distance from the ship to the island is approximately 26.10 miles.

2. The perimeter of a field is surveyed. The field has four sides. The lengths of the sides and the angles between them are measured as follows:

Side a length 54.507 meters, angle from side a to side b 117.68 degrees.

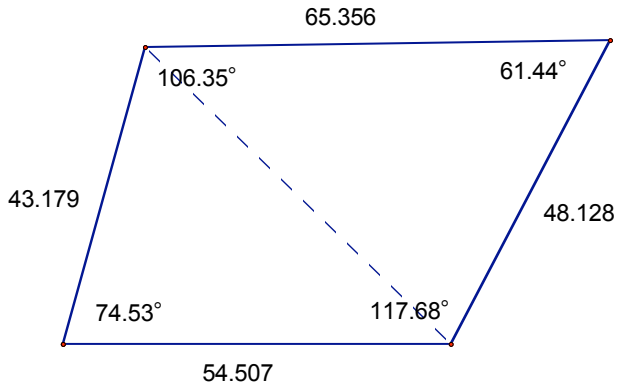
Side b length 48.128 meters, angle from side b to side c 61.44 degrees.

Side c length 65.356 meters, angle from side c to side d 106.35 degrees.

Side d length 43.179 meters, angle from side d to side a 74.53 degrees.

What is the area of the field?

We divide the quadrilateral into two triangles as indicated in the diagram:



The area of the quadrilateral is the sum of the areas of the two triangles, and we compute these areas using the sine area formula:

A = 1/2(43.179)(54.507) sin(74.53°) + 1/2(48.128)(65.356) sin(61.44°)
= 1134.144468 + 1381.352594 = 2515.497062

The area of the field is approximately 2515.50 square meters.

3. (a) What is the relation between the angles of a triangle and the area of a triangle in hyperbolic geometry and in spherical geometry?

Let T be a hyperbolic triangle. If the sum of the angles of T is S and the area of T is A, then $S < \pi$, and $\pi - S = A$. On the other hand, if T is a spherical triangle with angle sum S and area A, then $S > \pi$, and $S - \pi = A$.

(b) What is the relation between the angles of a polygon and the area of a polygon in hyperbolic geometry and in spherical geometry?

Let P be a hyperbolic polygon with n sides. If the sum of the angles of P is S and the area of P is A, then $S < (n - 2)\pi$, and $(n - 2)\pi - S = A$. On the other hand, if P is a spherical polygon with angle sum S and area A, then $S > (n - 2)\pi$, and $S - (n - 2)\pi = A$.

(c) A sphere is divided up into congruent spherical quadrilaterals. Each of the quadrilaterals has two angles equal to $\pi/2$ radians and two angles equal to $2\pi/3$ radians. How many quadrilaterals are there?

The sum of the angles of one of the spherical quadrilaterals is $S = \pi/2 + \pi/2 + 2\pi/3 + 2\pi/3 = 7\pi/3$. So the area is $S - (n - 2)\pi = 7\pi/3 - 2\pi = \pi/3$. If the number of quadrilaterals is m, then m times the area of a quadrilateral is the area of the sphere, or $m(\pi/3) = 4\pi$, so m = 12. (An example of such a spherical polyhedron is the *rhombic dodecahedron*.)