

In the following proofs, you may use our axioms, basic theorems, useful theorems, or any theorem proved in class or in the homework. Clearly identify any theorem that you use.

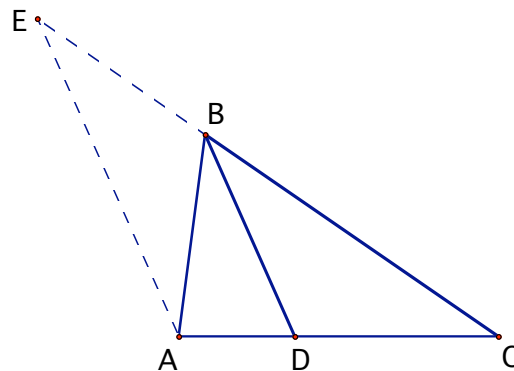
1. (50 points) Prove the following theorem. Write your proof in complete sentences, and give a reason for each step in your proof.

Theorem: If a triangle has the same circumcenter and incenter, then it is an equilateral triangle.

2. (50 points) Give reasons for each of the numbered steps in the following proof outline. The reasons for each step can involve one or more axioms or theorems, together with one or more previous steps in the proof outline.

Theorem: The bisector of an angle of a triangle divides the opposite side into segments that are proportional to the adjacent sides.

Proof. We show that if ABC is a triangle, then the bisector of the angle ABC intersects the side AC at a point D such that $AD/DC = AB/BC$.



1. Angle ABD equals angle DBC.
2. There is a line L through A parallel to the line BD .
 - Let E be the intersection of the line BC and the line L .
3. Angle ABD equals angle BAE.
4. Angle AEB equals angle DBC.
5. Angle AEB equals angle BAE.
6. $EB = AB$.
7. Angle AEB equals angle AEC.
8. Triangle AEC is similar to triangle DBC .
9. $AC/DC = EC/BC$.
 - $AC = AD + DC$ and $EC = EB + BC$, by the segment addition theorem.
10. $AD/DC = EB/BC$.
11. $AD/DC = AB/BC$.