

MATH 5200/7200 Final Exam, Dr. McCrory
December 12, 2008, 12:00 – 3:00 pm

For problems 1, 2 and 3, you may use our axioms, basic theorems, useful theorems, or any theorem proved in class or in the homework.

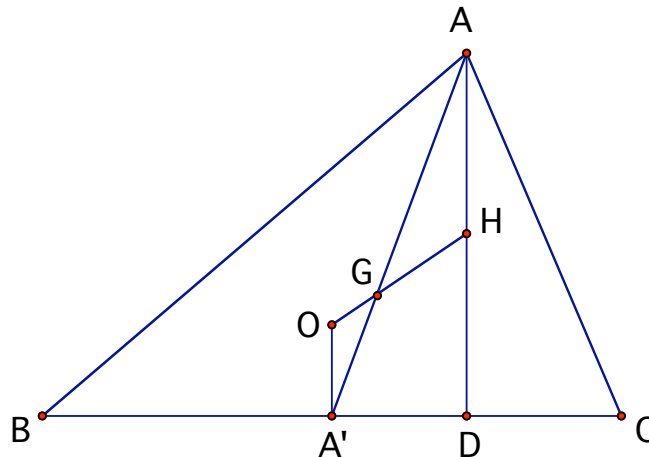
1. (15 points) Given a segment AB and a line L , explain how to construct a point C on L such that ACB is a right angle, using only the Greek construction rules. Prove that your construction works.

2. (15 points) Prove the following theorem. Write your proof in complete sentences, and give a reason for each step in your proof.

Theorem: If the diagonals of a quadrilateral $ABCD$ intersect at a point E , and $AE/EC = BE/ED$, then $ABCD$ is a trapezoid.

3. (20 points) Give reasons for each of the steps in the following proof outline. The reasons for each step can involve one or more axioms or theorems, together with one or more previous steps in the proof outline.

Theorem: For every triangle ABC , if the circumcenter O and centroid G are not the same point, then the orthocenter lies on the line OG .



Proof:

1. Let A' be the midpoint of BC .
2. G lies on the segment AA' .
3. OA' is perpendicular to BC .
4. Let H be the point on the ray OG such that $OH = 3(OG)$.
5. $GH = 2(OG)$.
6. $GA = 2(A'G)$.
7. AH is parallel to $A'O$.
8. AH is perpendicular to BC .
9. Similar arguments show that BH is perpendicular to AC and CH is perpendicular to AB . Thus H is the orthocenter of ABC .

For problem 4, use a calculator to compute your answer. Draw a diagram and show all the steps in your computation.

4. (20 points) Two fire towers A and B are 12 miles apart. Viewed from tower A, tower B has compass bearing S 65° E. A forest fire is spotted from towers A and B. From tower A the fire is N 60° E. From tower B the fire is N 45° E. How far is the fire from each of the two towers?

5. (15 points) A sphere is divided up into congruent spherical triangles. Each of the triangles has one angle equal to $\pi/2$ radians and two angles equal to $\pi/3$ radians. How many triangles are there? Explain your reasoning.

6. (15 points) Questions on class presentations:

(a) How do you measure the distance between two points in taxicab geometry?

(b) What are the Fibonacci numbers?

(c) State Napoleon's Theorem.

(d) Why were ancient Egyptian engineers called "rope stretchers"?

(e) How do you make a parabola using paper folding?