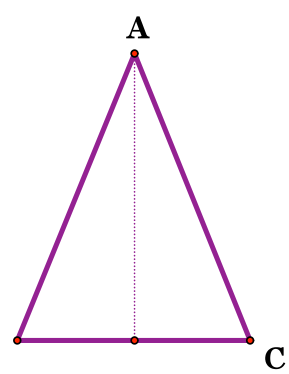
Basic Theorem Proofs

*** It is very important to prove in a right order to make the theorems work together!!!**

Set 1

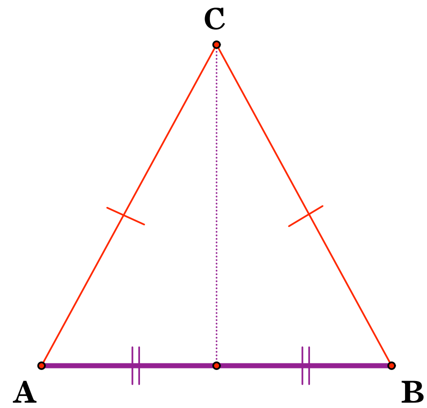
1. Isosceles Triangle Theorem (Part I)
   : When the points A, B, C are noncollinear, if AB = AC, then a(CBA) = a(BCA).



   1) SQ: Do they allow using SAS congruence?
      IA: Yes, SAS congruence is an axiom.
   2) IQ: Why ray AD intersects with BC?
      IA: Since ray is between ray AB and ray AC, by the betweenness axiom, ray AD intersects with BC.
   3) SQ: What if A, B, and C are on the different plane?
      IA: In Euclidean geometry, we consider only one plane.

2. Perpendicular Bisector Theorem (Part I)
   : If a point lies on the perpendicular bisector of AB, then the point is equidistance from A and B.
   - Comment:
     1) By the definition of line and the line axiom, we can construct a midpoint.
     2) We have to prove that "a point on the line segment can form a perpendicular line" first.

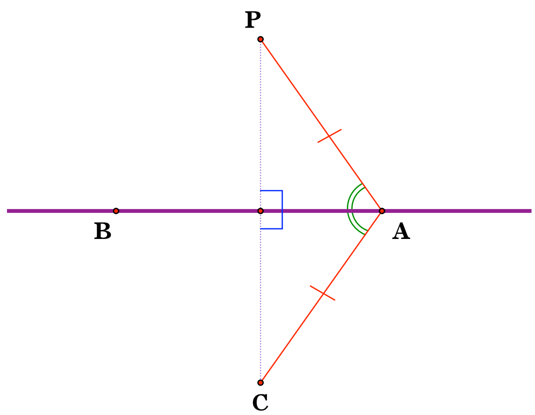
3. Perpendicular Bisector Theorem (Part II)
   : If a point is equidistance from A and B, then the point lies on the perpendicular bisector of AB.



1) We Know d(C, A) = d(C, B)
   We Want C lies on perpendicular bisector of AB
2) Proof: We get equal base angles by 1st half of Isosceles Triangle Theorem.
   Construct a median and connect to C.
   Then, by SAS congruence axiom, ∠CDA = ∠CDB.
   By the Straight Angle Theorem, ∠CDA + ∠CDB = 180 degrees.
   So, ∠CDA = ∠CDB = 90 degrees.
   So, CD is a perpendicular bisector.
   So, C is on a perpendicular bisector of AB.

4. Isosceles Triangle Theorem (Part II)
   : When the points A, B, C are noncollinear, if a(CBA) = a(BCA).
   1) SQ: Why PB = PC?
      IA: It is hard to prove by contradiction using GSP because GSP doesn't lie. ^^
   2) IQ: Why can we find P such that PB = PC?
      IA: By the Perpendicular Bisector Theorem, we can construct P.
   3) Proof: ∠ABC = ∠ACB + ∠ABC = ∠ACB
      So, ∠ACP = ∠ACB - ∠PCB = 0 degree by the Angle Addition.
      Since P is on the ray AB, A = P.
      * This proof is not using "contradiction"!!!

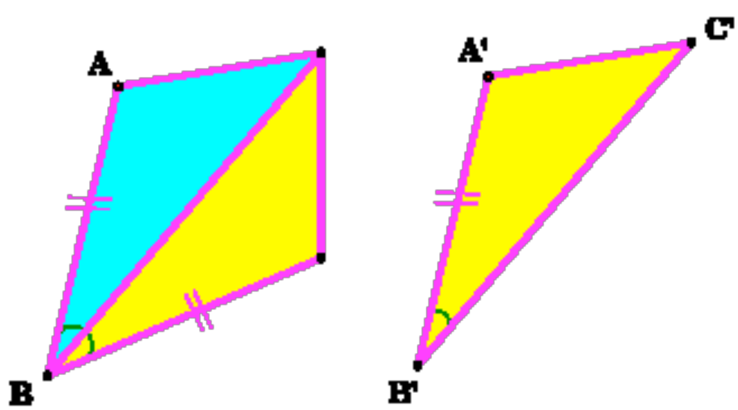
5. Existence of Perpendicular Line Theorem



Proof Pick any point on the line and construct a line segment.
Then we can copy the angle ∠BCA on the other side of the line.
If we connect P and C, ΔPDA ≈ΔCDA.
So, ∠PDA = ∠CDA.
By the Straight Angle Theorem, ∠PDA + ∠CDA = 180 degrees.
So, ∠PDA = ∠CDA = 90 degrees.
So, line PC is the perpendicular line of line AB.

Set 3

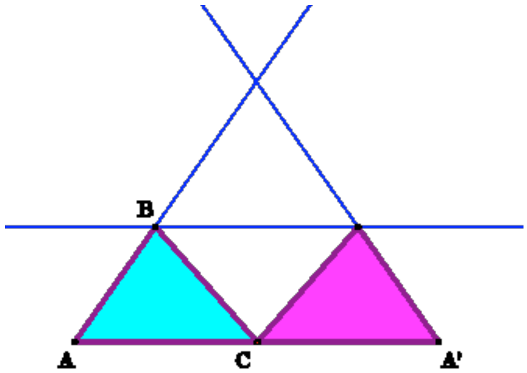
1. SSS Congruence Theorem
   - Comment: In this proof, they moved triangle, but there is no axiom about moving triangles. So, instead of moving a triangle, construct a triangle next to each other by using measurement.



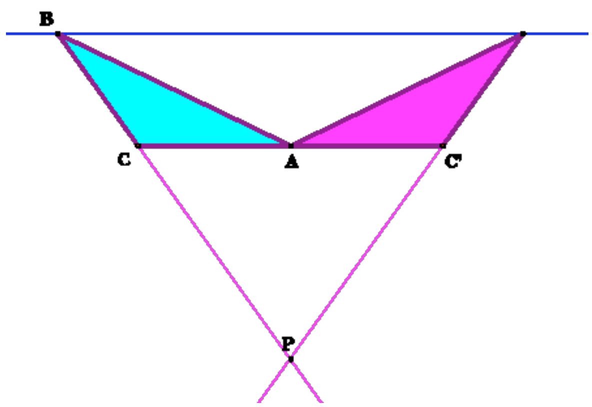
➔ First, measure same angle.
Second, cut off by measurement.
Third, by SAS, we can construct the congruent triangle.

2. ASA Congruence Theorem

<u>Case 1</u>

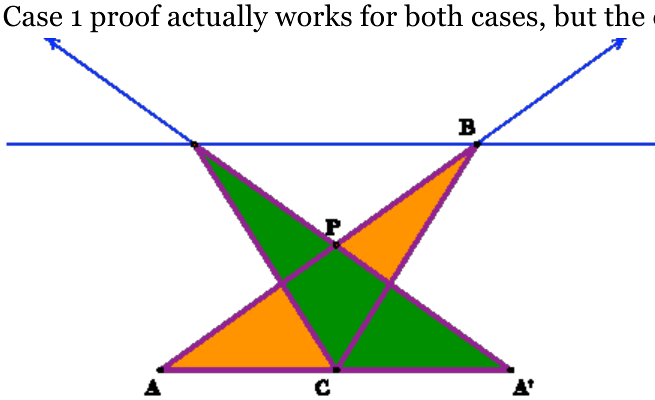


<u>Case2</u>



1) IQ: How do you know ray AP and ray A'P are intersecting?
   SA: Parallel Axiom(← if 2*∠PAA' < 180 degrees, then ray AP and A'P should intersect)
2) SQ: Where were we given the ∠PAA' < 90 degrees?
   IQ: It's from the case. Case1 was "Not Obtuse"

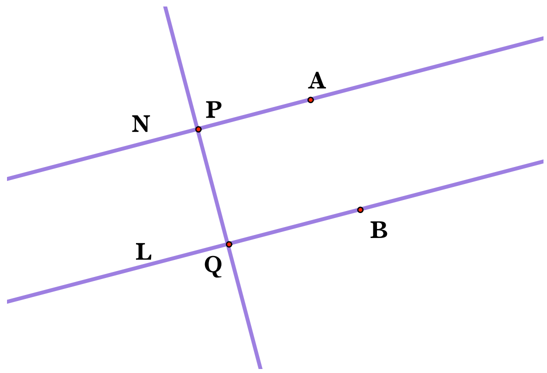
- Comment:
  a. Case 1 proof actually works for both cases, but the diagrams are very different.



b. Use easier diagram for "Communication," not for logical way.

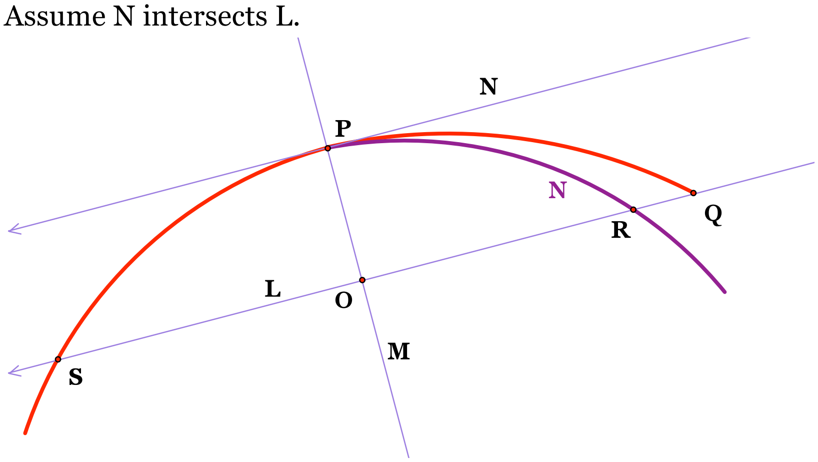
Set 2

1. Existence of Parallel line Theorem
: If I have a line L and a point P is not in the line, I have to prove line M that is parallel to L and passes P.



Proof

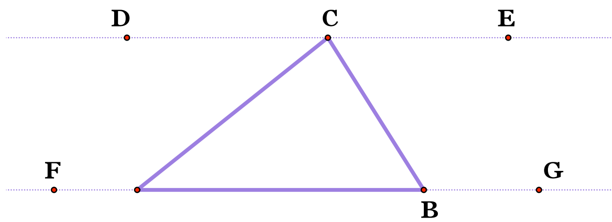
1) We can construct perpendicular M through P to L.
2) Construct perpendicular N through P to M.
The axiom says, "If ∠APQ + ∠PQB < 180 degrees, then they'll intersect"; however, Euclid didn't say about this case.
So, we have to use "contradiction."
3) Assume N intersects L.



Then, there exists Q further than R by the line axiom.
Connect Q and P.
Then, ∠QPO is greater than ∠RPO, 90 degrees.
So, ∠OPS < 90 degrees.
Then, by Euclid Axiom, N' intersects L.

This is contradiction by uniqueness of a line (Line axiom), i.e. line L' intersects at two different with line L.

2. Triangle Sum Theorem
: The sum of the interior angles of a triangle is 180 degrees.



Proof

Construct a parallel line through C to AB.

Then, ∠DCA = ∠CAB and ∠ECB = ∠CBA by the Corresponding Angles Theorem.

By the Straight Angle Theorem, ∠DCA + ∠ACB + ∠ECB = 180 degrees.

So, 180 degrees = ∠DCA + ∠ACB + ∠ECB = ∠CAB + ∠ACB + ∠CBA.

Since ∠CAB, ∠ACB, and ∠CBA are the interior angles of triangle ABC, the sum of the interior angles of triangle ABC is 180 degrees.

- Comment:
  1) In Euclidean geometry, the 'existence of parallel' and the 'sum of the interior angles of a triangle is 180 degrees' are closely related.
  2) In Hyperbolic geometry (Non-Euclidean geometry),
     the sum of the interior angles of a triangle ≠ 180 degrees &
     there exists more than one parallel line to a line.

Set 4

1. Area of Parallelogram
   1) SQ: How did you get the w*h = (w – b) * h + A?
   2) SQ: How did you get angles are congruent?
   - Comment:
     a. Interestingly, this proof did not use the area of triangle.
     ➔ Because they can use only the area of rectangle.
     b. Decomposition of area
     c. This proof works for every case because it didn't use the height.

2. Area of Triangle
   - Comment:
     1) Now, we can use the area of parallelogram.

2) A triangle has 3 sides, but we don't need 3 different proofs because there is nothing about the shape of the triangle.

- Corollary: The proof can be used for any side of the triangle.
- ½ bh is true for 3 different sides.

Set 5

1. AA Similarity
   1) IQ: What kind of similarity did they use?
      SA: SAS similarity.
   2) IQ: What kind of congruence did they use?
      SA: ASA congruence.
   3) SQ: How can you use ratio if you can't use measurement?
      IA: We can use the measurement. It is part of axiom (→ consequence of line & distance axiom)

2. SSS Similarity
   1) SQ: Extend AB → make B" such that AB"/AB = r
      Construct BC // B"C"
      Then ∠B" = ∠B.
      So, triangles ABC and AB"C" are similar triangles by ASA similarity.
      So, AC"/AC = r and B"C"/BC = r.
      So, triangles A'B'C' and AB"C" are congruent by SSS congruency.
      So, triangles ABC and A'B'C' are similar.