Math 5200/7200 Geometry
Friday, October 24th, 2008

Review of problem 1 from Test #2. Most people made the same mistake; we need to learn from this.

MISTAKE: Assume that every angle bisector is a perpendicular bisector of the opposite side.
The hypothesis actually says that the two points, the incenter and circumcenter, are the same; the hypothesis says nothing about lines. It's important to go back to the definition, not the construction.

Incenter is equidistant from the side of the triangle.
Circumcenter is equidistant from the vertices of the triangle.
To find, **distance from incenter to the side** we must drop a perpendicular from the incenter to the side.

Brief Outline of proof: We have six right triangles, use Pythagorean theorem to show that all three sides the same, then the sides of the original triangle are the same.

McC: Thought this was an easy problem, because I didn't foresee that people would make this false assumption.
Eric: Can you use 30-60-90 triangles?
McC: That is circular reasoning—would have to prove that its equilateral to get that it is 30-60-90 triangles
McC: Purpose of this Can you assume things that aren't given, give something familiar to see if you can step back and not make assumptions beyond what was given. However, this was more extreme than I had intended. Solutions will be posted on the website.

Problem 2 for test 2 is interesting but not any "tricks"

TRIG!!!
Emphasize relationship of trig to geometry; there is a gap between geometry and trig in schools!
We cover some theory followed by some applications.
Detour from syllabus to introduction to non-Euclidean geometry.
Guest lecture from next semester professor

Link on the website to definition of trig functions. Only focusing on sine and cosine. Best to know these before moving to others. The definition is based on Angle-Angle Triangle Similarity (using internal ratios instead of going between similar triangles). Sine and Cosine only depend on the measure of the angle.

What if you have other angles? (not the acute angles of a right triangle) To deal with this use the definitions provided in calculus course— essentially making coordinate system. Be sure to keep track of positive and negative angles.

Bunch of great properties follow from these two definitions, and they are included on the webpage.
Michelle: Do we need to prove these properties?
McC: Since these are direct consequences of the definition, you can just use them. You should prove them for yourselves, I will not prove them in class. It will take some organization to see how this works.

Trig was developed after Euclidean Geometry because it involves measurement, which was not considered by Greeks. Came out of applications to astronomy. (For example, 360 degrees corresponds almost to number of days in the year, why 360 instead of 365, because 360 has lots of divisors—astronomy isn't totally exact—for example leap years – possible that a year used to be defined as 360 days?)

Homework for Monday:

The three most important properties of sine and cosine (beyond the basic ones):
Standard notation: Side opposite angle A is side a.

1) **Area:** half the product of two sides of the triangle, and the sine of their included angle. (This is generalization of special case for right triangles. Think about 2 cases, with acute and obtuse angles)

2) **Law of Cosines**: The square of one side of a triangle is equal to the difference of the sum of the squares of the other two sides and twice the product of those two sides and the cosine of their included angle.
(Again, 2 cases to consider, but this one is tougher!!!)

3) **Law of Sines**: In a triangle, the ratios of the length of a side to the sine of the angle opposite that side are constant. (2 cases again)

Homework #14—see the web.

We're focusing on: Standard applications, trying to give ideas for teaching, how to use trig functions to simplify proofs in geometry. Angle addition formulas coming next week.

<u>**Monday, October 27, 2008**</u>

<u>***Lindy presents #1: Area of a Triangle.***</u>

Adam: Did she go "backwards?" She showed the formula ½ ab sin(C) is equal to area formula not derive it the other way.

McC: This is a good formula; You don't actually the construction line that gives the diagonal distance in practice. So there could be a lake or mountain or something in the middle that keeps you from being able to measure this distance.

<u>***Laura presents #2: Law of Cosine***</u>

Case 1: <C is obtuse….
Must use property: Cosine an angle is the opposite of the cosine of the supplement.

Adam: Can you do this in one case?
McC: No, we can't do this in one case. Its important that cos changes from being negative to positive…
Knowing what you are trying to get will help you figure out how to structure the proof, you know you need cosine C, so what is that in this diagram and how does it relate to the other things you need?

<u>***Mary Catherine presents #3: Law of Sines***</u>

Her proof covers both acute and obtuse case.
Allyson: Why?
McC: Because height is same if the angle is acute or obtuse
Our goal here is to see things from fresh viewpoint, so we can close the gap between geometry and trig.
Some might say sine of angle and its supplement are the same and use that the supplement's sine is h/b. This falls back on properties. It follows from our definition that sine of obtuse angle is h/b, you can also prove the properties listed on the definition page and use these.

Looking at definition:

Using similar triangles can scale any triangle down so that hypotenuse is 1, and use unit circle, the defining cosine as x coordinate of intersection of terminal ray of angle (where initial ray of angle is positive x-axis) and unit circle.

How can we prove from this definition that angle and its supplement have the same sine?
Figure out way to get supplement with initial ray on pos. x-axis.
Intersection of unit circle with line that does thru initial point that is // to x-axis…now, one angle is supplement of the other, then use given definition to show that sine of angle and its supplementary angle are the same.

You can use the same to show the corresponding property for cosine. Cosine of angle and cosine of its supplement are opposite.

Similar construction to show properties for negative angles. Sin of angle is opposite of sine of negative angle. Cosine of angle is equal to cosine of negative angle.

How can you construct the complement? Reflect over y = x… (*Thanks to Adam and his gre studying*)
This actual construction looks pretty complicated; the upshot is: sine of an angle is the cosine of the angle complement.

Pedagogical point from McC: I hate trig because none of the formulas were ever explained. You just have to learn them, why aren't the formulas explained, because of the disconnect between trig and geometry.

Other applications:

1) Relation between area formula and law of sines
   Useful to realize that sin area formula is three different fomulas. Set these equal and low and behold, you get law of sines. How lovely.

2) Congruence theorems and trig formulas.
   SAS congruence, practical way to think of this: if you given me an angle (included) and two lengths that uniquely determines the triangle. The opposite side is interesting' what formula gives you this—law of cosines-- shows exactly how the opposite side is determined from SAS. How could I get the other two angles? Law of sines: Main thing you used to learn in trig: soving trinalges.

Ran out of time to discuss ASA and SSS

<u>Wednesday, October 29, 2008</u>

Go over homework questions due next Monday:
#1) Website for compass bearing (which has a lot of other fun stuff in it)
Always north or south, then degrees to the east or west. (Yes, southeast is 45 degrees east of south.)

Compass reading is good application for the elementary kids, thought maybe today's kids need GPS applications.

#2) tenth of a degree is tiny….pretty sure you can't measure more accurately than a tenth of degree.

#3) use law of cosines

#4) no comments

#5) you may want to explore this with gsp to make sure you believe it before you attempt to prove it. Will use law of sines.

Applications of trig

1) Rotation matrices

$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ 1st column moves <0, 1> vector to $\langle \cos\theta, \sin\theta \rangle$,

2nd column creates perpendicular to that new vector

$$R_\phi R_\theta = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\phi\cos\theta - \sin\phi\sin\theta & -\cos\phi\sin\theta - \sin\phi\cos\theta \\ \sin\phi\cos\theta + \cos\phi\sin\theta & \cos\phi\cos\theta - \sin\phi\sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) \\ \sin(\theta+\phi) & \cos(\theta+\phi) \end{bmatrix}$$

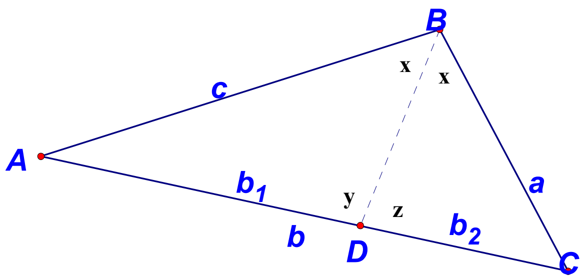
2) Complex numbers

$z = x + iy = r(\cos\theta + i\sin\theta)$ and $w = u + iv = s(\cos\varphi + i\sin\varphi)$

then $zw = (x+iy)(u+iv) = (xu - yv) + i(uy + xv) = rs(\cos\theta\cos\varphi - \sin\theta\sin\varphi) + i(\cos\varphi\sin\theta + \cos\theta\sin\varphi)$

And these are sine and cosine addition laws! So $zw = rs[\cos(\theta+\varphi) + i(\sin(\varphi+\theta)]$.

3) Proofs using trig, for example problem 2 from the exam



Need to show $\frac{b_1}{b_2} = \frac{c}{a}$. We'll use th law of sines. $\frac{\sin x}{b_1} = \frac{\sin y}{c}$ and $\frac{\sin x}{b_2} = \frac{\sin z}{a}$

sin y = sin z since y and z are supplements (fron the application of straight angle theorem).

$\frac{c \cdot \sin x}{b_1} = \sin y$ and $\frac{a \cdot \sin x}{b_2} = \sin z \rightarrow \frac{a \cdot \sin x}{b_2} = \frac{c \cdot \sin x}{b_1}$

and then $\frac{a}{b_2} = \frac{c}{b_1} \rightarrow \frac{b_1}{b_2} = \frac{c}{a}$!!!