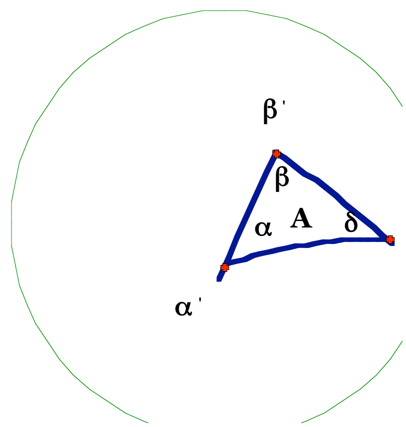
Notes for 5200/7200
November 19, 2008

- Dr. M asks if there are any questions on the homework #1
    - You can turn it in class on paper
    - Very open ended
    - Thought Experiment
- Answer question from Allison from last class
    - Sum of angles is always bigger than 180 (π)
    - The angle excess – π = area
    - As the area gets smaller, the angle excess gets smaller (closer to 180)
    - The largest would 5 π
    - There is a way to get triangles to fill up a whole sphere, this is where Spherical Geometry is very different than Euclidean Geometry
    - Drawing a Geodesic triangle actually produces two triangles (make one really small, then the other will have area approaching 4π)
    - Another check on our theorem is below:



Area of Triangle A $=\alpha+\beta+\delta-\pi$

Then, what is the area of triangle A'?

Area of Triangle A' $=\alpha'+\beta'+\delta'-\pi$

$$(2\pi-\alpha)+(2\pi-\beta)+(2\pi-\delta)-\pi = 4\pi - A$$

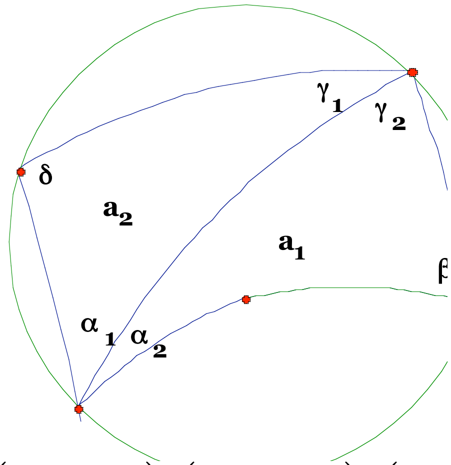
$$6\pi-(\alpha+\beta+\delta)-\pi =$$

$$5\pi-(\alpha+\beta+\delta)=$$

$$5\pi-(A+\pi)=$$

$$= 4\pi - A$$

- What about other polygons on a sphere?
    - Divide them into triangles



$$\alpha + \beta + \gamma + \delta = (\alpha_1 + \delta + \gamma_1) + (\alpha_2 + \beta + \gamma_2) = (a_1 + \pi) + (a_2 + \pi)$$

$$\alpha + \beta + \gamma + \delta = a + 2\pi$$
$$\alpha + \beta + \gamma + \delta - 2\pi = area$$

- If we wanted to find the area of a 5-sided shape, we would need to make 3 triangles. If we wanted to find the area of 6-sided shape, we would need to make 4 triangles.
- The formula for the area of a polygon with n-sides

$$\sum of\ the\ angles - (n-2)\pi = area$$

- Discussion of Homework Question 1
  - It is an essay question
  - Look at axioms and basic theorems for spherical geometry
  - Go through axioms one by one and see if they work in spherical geometry
  - Angles measured in the same way as in the plane
  - Area and distance is same
  - We know what are three basic measurements are
  - We have
    - Distance
    - Line
    - Angle
    - Betweenness
    - SAS congruence
    - Parallel Axiom
      - Every two line intersect at two points
      - Since it doesn't hold, explain why and what would work in spherical geometry
      - Put in as much information as you can
    - Area
      - Formula is suspicious, there are no rectangles, because you can't have 4 right angles, there would be no angle excess and zero area

    - This is not an internet research question, plus it will be difficult to use because not necessarily a standard axiom hold
    - Don't just say "it holds" give him reasons as to why it holds
- On Friday, we will talk about spherical trigonometry, much simpler than in Euclidean Geometry, and Lenart Spheres
- Isometries in Spherical Geometry
    - Remember in GSP there are certain transformations that are isometries
        - Basic ones are translations, reflections, rotations
        - In Euclidean Geometry, many transformations preserve distance except dilation
        - The isometries in Spherical are easy to understand
            - Correspond to congruence of triangles
            - Rotations of the sphere and you can do that with any pair of poles (endpoints of a diameter)
            - There is also reflection
                - Easy to understand using the equator
                - You can do reflection with any geodesic
            - You can combine by rotating and reflecting
            - The characteristic feature of isometry is it preserves distance and congruence