Notes 5200/7200
November 21, 2008

- Let us discuss the second problem from homework 19.
  - Jenny will discuss how she worked the first problem.
  - Since 3 angles come together at a vertex, and we know the sum of the total angles of a circle
  - For the dodecahedron:

The sum of the angles $= \frac{2}{3}\pi(5) = \frac{10}{3}\pi$

We know the area of the sphere is 4π.

Since there are 12 congruent faces, we may say

$$4\pi \div 12 = \frac{\pi}{3}$$

The formula for angle excess is $\sum \textit{of the angles} - (n-2)\pi = \textit{area}$

$$\frac{10}{3}\pi - (5-2)\pi = \frac{\pi}{3}$$

This worked, for $\frac{\pi}{3}$ is the angle excess

  - For the icosahedron:

The sum of the angles of a circle is 2π

Divide 2π by 5, because there are five angles at one vertex

The sum of the interior angles of the triangle is

$$\frac{2}{5}\pi(3) = \frac{6}{5}\pi$$

We know there are 20 congruent faces and the area of a sphere is 4π

Thus, the area of each face should be $\frac{4}{20}\pi = \frac{\pi}{5}$

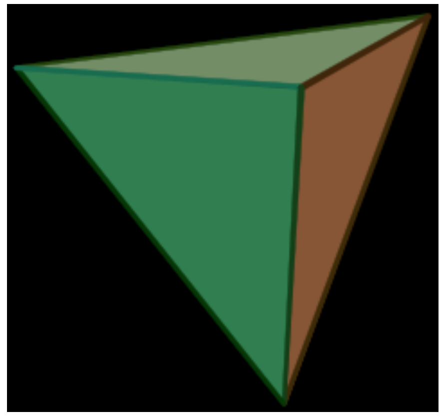
Check using the angle excess formula:

$$\sum \textit{of the angles} - (n-2)\pi = \textit{area}$$

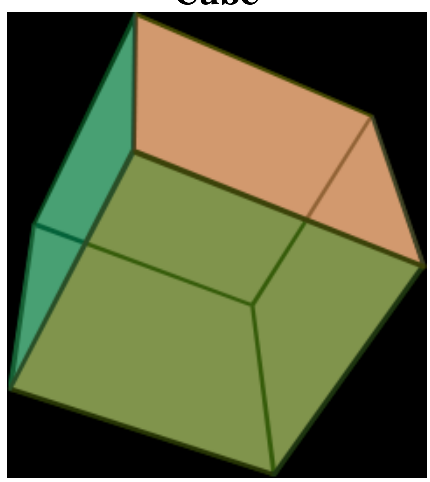
$$\frac{6}{5}\pi - (3-2)\pi = \textit{area} = \frac{\pi}{5}$$

- For the exam on December 3, we will be accountable to know the five platonic solids

Tetrahedron

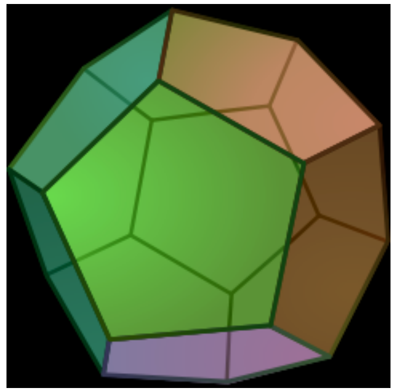


Cube

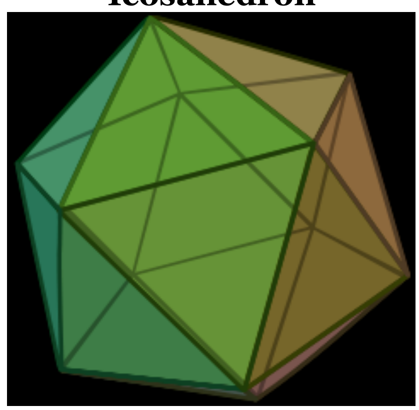


Octahedron

Dodecahedron



Icosahedron



- Discussion of the axioms

(1) Distance axiom: Distance is symmetric, positive definite, and satisfies the triangle inequality.

In Spherical Geometry, you have two geodesic segments. The triangle inequality is about distance in spherical geometry, you always take the shorter of the two geodesic arcs.(Note the longest distance two points can have is half of the circumference). The triangle inequality refers to the shortest distance, and therefore does hold in Spherical Geometry. DISTANCE IS THE SHORTEST GEODESIC DISTANCE.

In Spherical we need a term for the sequence of arcs and a term for the region (as the sequence of arcs determines more than one region). We need a name for the series of sequences, so that when we say "triangle" we mean area. We need to specify if it is the triangular region verses the arcs that make up the triangle.

A triangle can be a "straight" if the three point are on the same line. It is the degenerate example.

Distance is positive definite in spherical geometry, and the triangle inequality also holds in spherical geometry.

(2) Line axiom: Two points determine a line. There exist three points which do not lie in a line.

It is true that three points do exist that do not lie on a line in spherical geometry.

A line is not infinite. We need to parameterize the line, and say the length of the great circle is 2π *if the radius is 1.*

Segment is an arc. Two points determine at least two segments. If they are antipodal points, they determine an infinite number of segments.

We do not have a ray in spherical geometry. If we want to talk about angles, we will use arcs. (A possible definition of a ray in Euclidean is half of a line, so in Spherical we could define a ray is half a great circle, where it starts at a point and approaches its antipodal- if you leave in the antipodal point then it would seem to have two starting points)

(3) Angle axiom: The angle function is antisymmetric, it depends only on rays, and it is additive. Angles of every measure occur with a given ray as initial side.

The basic thing is you have to have an angle determined by three points, it should work. What If A and B are antipodal to O? The angle would be zero. McCrory Ok, I guess…Seems ok. It is really a Euclidean angle measure.

(4) Betweenness axiom: Betweenness for points on a line corresponds to betweenness for rays from a point.

The definition of betweenness, where C is between A and B is not good in spherical. Every point is between A and B. A and B have two segments between them.

First let us define it when A and B are not antipodal. The point C is between A and B if it lies on the minor arc.

Now, what if A and B are antipodal, then every point C is between A and B. Like, every city on the earth is between the North Pole, A, and the South Pole, B.

David Hilbert said you really need to talk about 4 distinct points on a great circle. There is only one way to separate them into pairs, so that two are specifically "between".

Consider two couples on a date. To understand Hilbert's idea of between, so that the couple separates the other pair.

Betweenness should be fine with angles, because we define consistent notation for direction. We can still talk about a ray being between two rays using Eric's idea for rays.

Suppose we take our definition for between to mean the C is between A and B if C is on the minor arc of A and B.

If we use betweenness to be the shortest segment between two points, it should

The last three were discussed in class on Wednesday.

Now we can see why spherical geometry is not the same as Euclidean, because certain ideas that are very basic in Euclidean are not similar to Spherical.