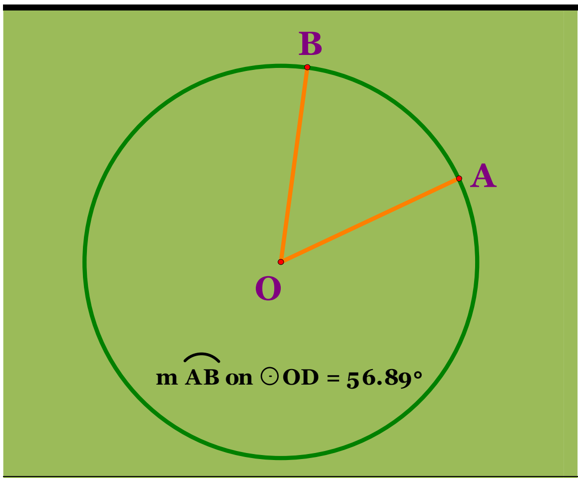
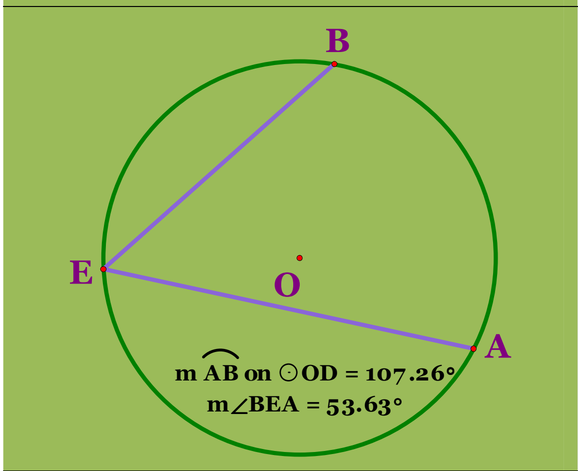
HW Review

1. Relation between the angles of a triangle and the arc angles of the circumcircle

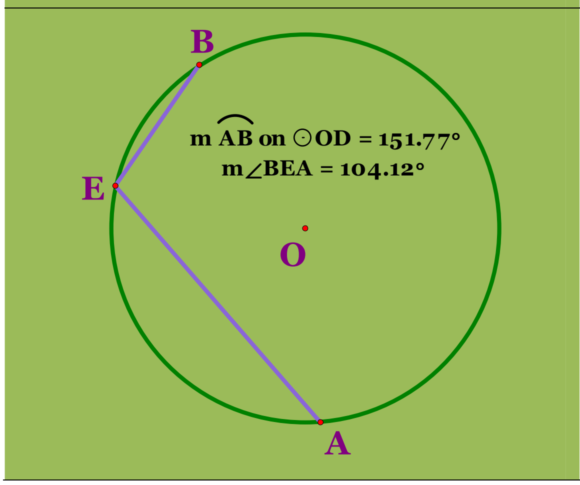


a. Select two points A & B, and circle O
b. Go to "Measure" and click "Arc Angle"

- 0° < angle BOA < 180°
- Since m(arc AB) on circle OD = 56.89°, we can say that GSP always measures "Minor Arc"

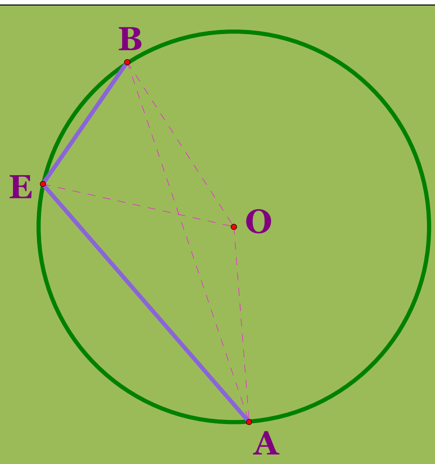


- m (angle AEB) = 53.63°
- CALCULATE:
  360° - m (arc AB) = 107.26°
  m (angle AEB) · 2 = 107.26°

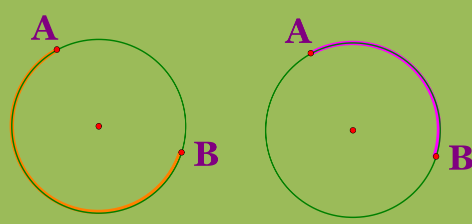
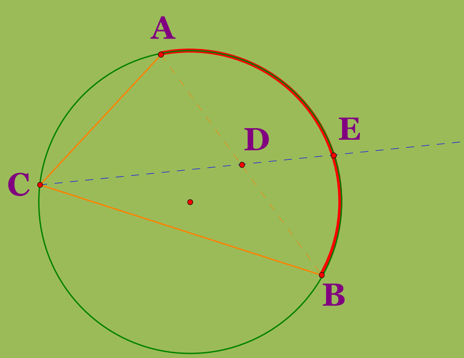


- m (arc AB) on circle OD = 151.77°
- m (angle AEB) = 104.12°

  So,
  m (arc AB) OR 360 – m (arc AB) = m (angle AEB) · 2

	▪ If acute angle, m (minor angle) = 2 · interior angle ▪ If obtuse angle, m (major angle) = 2 · interior angle
Conclusion	This holds if we always measure the arc that doesn't contain the vertex.

2. How to force GSP to measure major arc?

1. Use the construction property of GSP 	▪ GSP always construct an arc "counter-clockwise" ➔ If you choose point A and then B (of course, the circle, too!!!), GSP would construct major arc AB; if you choose point B and then A, GSP would construct minor arc AB ➢ Doesn't always work!
2. Choose three points  m∠ACB = 65.15° m∠ACB·2 = 130.31° m $\overset{\frown}{AEB}$ = 130.31°	▪ Construct an arc with points A, E, and B a. Select points A and B b. Go to "Construct" and click "Midpoint" c. Select points C and the midpoint in order d. Go to "Construct" and click "Ray" e. Construct a point on the intersection of the ray & the circle and label as E f. Select points A, E, and B g. Go to "Construct" and click "Arc Through 3 Points" ➢ This method always works!!!

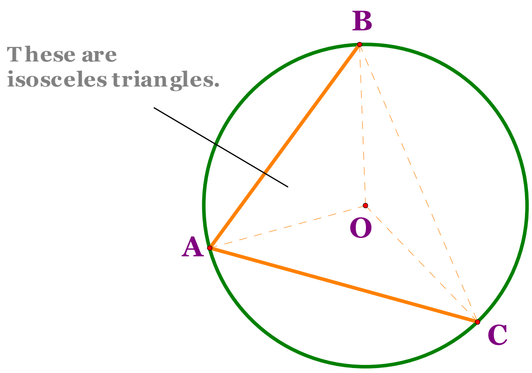
Animation	m(arc AB)=m(angle AEB)2_Ani.gsp

➢ **Conclusion**
Relation between the angles of a triangle and the arc angles of the circumcircle
1. **m (arc AB) OR 360 – m (arc AB) = m (angle AEB) • 2**
2. Construct an arc using GSP's construction property (counter-clockwise) → but NOT always work!
3. **Construct an arc with 3 points →** Always work!!!

3. Proof

Theorem

Arc-Angle Theorem
The arc angles of the circumcircle are twice of the angles of a triangle, respectively.

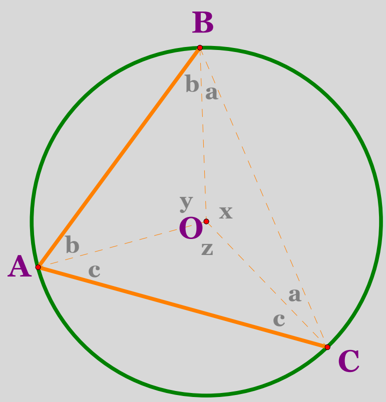


- Consider angle A and arc angle BC (= angle BOC)

✓ The first step of geometric proof is "adding some lines"

Proof

- When triangle ABC is an "Acute triangle"

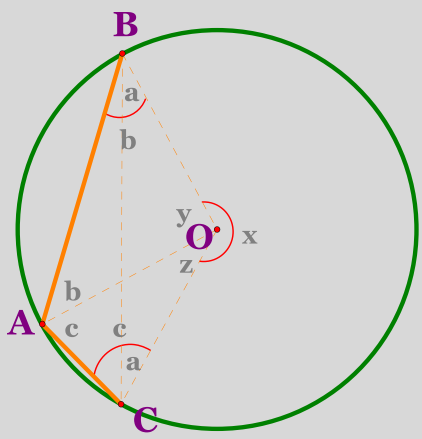


$x + y + z = 360$

x + (180 – 2b) + (180 – 2c) = 360
x = 2 (b + c)
Remind b + c = A
So, x = 2A.

✓ Even though our construction is sloppy, we can say that the triangle is an acute triangle if the center of the circle is inside of the triangle.

- When triangle ABC is an "Obtuse triangle"



x + y + z = 360
x + (180 -2b) + (180 – 2c) = 360
x = 2 (b + c)
Remind b + c = A
So, x = 2A.

✓ When you prove, you have to consider acute triangle and obtuse triangle separately!